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Contingent Worksharing

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Abstract

In a setting that focuses on efficient dynamic hours-workers substitution we show that contingent worksharing contributes to workers retention during bad business spells and to sustained hiring during good spells. As a consequence, average employment increases on both accounts. We also show that worksharing interacts with firing costs in affecting workforce decisions and determines the sign of the employment impact from an increase in firing restrictions.

Jel Classification: J23, J63, C61
Keywords: Temporary worksharing, Firing Costs, Stochastic methods

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1 Introduction

. This paper is concerned with the employment effect of contingent worksharing. For contingent worksharing we mean those arrangements that point to workers retention during bad business spells through temporary reductions in working hours as well as wages. Agreements between firms and workers representatives on contingent worksharing are frequent in Europe (IRS, 1999). This is due to a favorable institutional environment which provides means to prop up temporary losses of income for workers.

We study the employment impact of contingent worksharing in a partial equilibrium cost-minimisation model. In our setting, the representative firm operates in an uncertain environment and faces linear costs from adjusting labour both on the extensive (employees) and on the intensive margin (hours). The first are due to institutional hiring and firing costs, typical of many European labour markets. The second take the form of costs due to unbalances between actual working hours and the standard working time determined by institutions. There are overtime premia during spells of intense activity and payments for idle hours during periods of low production.

We show that, under our set of technical assumptions and in line with other contributions, the optimal employment policy is characterised by an *Ss* policy. As long as the intensity of production leads to unbalances between actual and standard hours that are contained within well defined boundaries, workforce inaction represents the optimal decision. Conversely, when unbalances move beyond these boundaries, workforce adjustments take place in order to reset actual hours onto the boundaries.

Using the optimal *Ss* policy we perform numerical comparative dynamics and find that contingent worksharing makes firms more reluctant to fire during periods of low production. Interestingly, we also find that worksharing makes firms more willing to hire during periods of intense production so that the impact of worksharing on employment is unambiguously positive. The force that drives both results is the increase in the net marginal value of employed workforce. Workers become more valuable to the firm if worksharing sets in when production is low. This contributes to their retention in bad times and to their recruitment in good times.

We also show that contingent worksharing affects the employment impact following an increase in firing costs. A well know result in the literature based on dynamic labour demand is that an increase in firing costs leads to an enlargement of the inaction space. The firm becomes more reluctant towards firing as well as toward hiring with an uncertain impact on the average employment level (Bertola and Bentolila, 1990). This result holds in the present paper too but with the following additional corollary. Due to worksharing, the impact on hiring and firing decisions is not symmetric. Reluctance towards firing increases more than reluctance towards hiring with the consequence that, following an increase in firing costs, average employment tends to increase.

Models of workforce adjustments have been formulated in several previous works[1]. None of these, however, analyses hours-employment decisions with worksharing and in a stochastic setting. Even if it is not concerned with the hours-employment substitution, the seminal work of Bertola and Bentolila (1990) represents the main reference for the techniques that have been used in the present paper. However, the paper contains elements of technical originality as it develops a simple and intuitive method to handle stochastic integration over discontinuous payoff functions. This method is inspired by the reading of Harrison (1985) and hinges on the evaluation of stochastic discount factors similar to Arrow-Debreu securities. To our knowledge, this approach is novel in the literature of reference.

The structure of the paper is as follows. Section 2 presents the setup without contingent worksharing. In section 3 we introduce worksharing and study its effects through numerical computations. Section 4 offers some conclusive remarks.

2 A problem of workforce management

2.1 Set up

We consider an establishment that produces a non-storable output flow y_t by using the following technology:

$$y_t = y(L_t, h_t) = L_t h_t \tag{1}$$

L_t represents the mass of employed workers and h_t the number of individual hours that are actually used for production purposes. Thus, by stating equation 1, we make three basic assumptions: production does not require capital, marginal returns for each labour component are constant and, related to the latter, workers can be employed for an unbounded number of hours. The absence of capital is explained by our focus on short term fluctuations which rule out capital adjustments from the set of available options. Constant marginal returns represent instead an innocuous simplification in the sense that all results can be obtained with a more general technology of the form $y_t = L_t^\alpha h_t^\beta$ $(\alpha, \beta \leq 1)$. Lastly, we are aware that the absence of bounds for the number of working hours is at odds with an obvious physical constraint and with the assumption of constant marginal returns. However, we find below that in equilibrium hours can not move outside a rather narrow corridor. This makes redundant any reasonable physical constraint and plausible the above assumption of constant returns.

The establishment is required to produce an output flow z_t which evolves as a (μ, σ)-geometric Brownian motion. That is, we assume that the firm that runs the establishment has a limited power or a limited benefit to smooth demand through price variations. This may be due, for

[1] See, for instance, Nickell (1986), Hamermesh (1989 and 1990), Bertola and Bentolila (1990) and Nuziata (2003).

instance, to high menu costs or to price-taking behaviour[2].

Non-storability implies $y_t = z_t$ at all times so that the firm has to adapt continuously L_t and/or h_t to respond to the output requirement. In line with the literature on workforce adjustment costs, we assume that to recruit and to dismiss a worker the firm has to pay a fixed hiring cost H and a fixed firing cost F. Hiring costs originate from advertisement, selection and training activities; firing costs are mainly due to provisions contained in the employment protection legislation.[3] To complete the description of the institutional setting, we use h^s and p for the statutory working time and for the over-time hourly premium and assume that both parameters are exogenous to the firm in that they are determined by legal provisions or by some economy-wide union-employers agreement[4]. As a consequence of this assumption, when h_t is larger than h^s, individuals work overtime and are entitled to perceive a wage premium. By contrast, if h_t is lower than h^s, the difference $h^s - h_t$ represents the amount of individual idle hours, i.e. hours that must be paid to the worker even if they are not effectively used in productive activities. This happens when workers stay at work for h^s hours but the tasks they perform could be done in a shorter period of time, we refer to this case as undertime.[5]

Finally, with w and r we represent respectively the exogenous hourly wage and the interest rate. The objective of the manager in charge of the establishment is the minimisation of the expected discounted stream of costs necessary to produce the flow z_t.

2.2 Fixed Workforce

To avoid overtime costs and undertime costs (i.e. payments for idle labour), effective working time h_t should be continuously pegged at the statutory level h^s. In turn, continuous pegging involves continuous workforce changes to adapt production to the output requirement z_t. Finally, continuous workforce changes lead to large firing and hiring costs due to the volatility of z_t. In the next subsection, we show that the optimal employment policy entails workforce inaction and free variations of h_t as long as the latter wanders within a corridor which contains h^s. In proximity of the lower boundary of the corridor undertime prevails so that firing would lead to a reduction

[2] The assumption that firms can not smooth demand through prices is common to many other works. See Galeotti et *al.* (2005) and papers cited therein, for instance. Marchetti and Nucci (2006) analyse a sample of Italian manufacturing firms and show that some firms use prices while some others use hours to respond to variations in demand. They interpret the behaviour of the latter as a result of high menu costs.

[3] With variable hours and, consequently, variable monthly wages, it may seem unwarranted to assume fixed firing costs. In fact, in many legislations, dismissed workers are entitled to perceive severance payments that are proportional to their most recent labour income. Yet, as reported in the introduction, in many European countries wage supplement schemes operate so as to keep workers incomes constant during periods of short hours employment. This makes severance payments virtually independent from the amount of hours worked under worksharing.

[4] See OECD (1998) ch.5 for a survey on the size of h^s and p across industrialised economies.

[5] " ... Undertime can take many forms, from hours spent away from the office on errands to chunks of time spent at your desk surfing the Internet...." Sue Shellenbarger, The Wall Street Journal, April 18, 2002

in undertime costs. The boundary is endogenously determined by a condition which equates the firing cost F to the expected saving in undertime costs following a dismissal. By contrast, close to the upper boundary overtime prevails. In this case, the boundary is determined by a condition which equates the hiring cost H to the expected saving in overtime costs following a recruit.

As a preliminary step to illustrate how to compute these boundaries, in this section we assume that H and F are so large that the expected saving in undertime and overtime costs is always smaller than the corresponding firing and hiring cost. That is, the firm does not find it profitable to dismiss a worker even if z and h are nil and it does not find it profitable to recruit a worker even if z and h grow unbounded. One may also regard this case as if the lower boundary were zero and the upper boundary infinity.

How large need F and H to be to prevent workforce adjustments in all demand conditions? If z and h are nil at current time they will be nil at all future times.[6] In this case, in the absence of workforce adjustments and with a workforce of size L, the firm pays a wage $h^s w$ to L unproductive workers. Thus, the overall discounted wage bill is $Lh^s w/r$. Dismissing the L workers would cost the firm FL so that it does not fire any of them if F is larger than $h^s w/r$. On the other hand, if z grows to infinity so do working hours h under the assumptions of constant marginal returns and absence of physical constraints. In this case, overtime becomes a permanent state and the firm does not hire if H is larger than $ph^s w/r$, i.e. if the cost of recruiting an extra worker is larger than the ensuing discounted saving in overtime costs.

The upshot of this discussion is that workforce is constant at all times if the following parameter restriction holds

$$\frac{h^s w}{r} < \min[F, H/p] \tag{2}$$

As a consequence of workforce being fixed at some level L, variations in z_t are accommodated only through changes in individual working time h_t: $h_t = z_t/L$. It follows that actual working time h_t evolves as a (μ, σ)-geometric Brownian motion. Let $S(h)$ represent the shadow value of workforce when h_t is currently at level h. Formally, $S(h)\, dL$ coincides with the *reduction* in the expected discounted flow of undertime/overtime losses if the firm, contrary to the implication of 2, increases its workforce by adding dL workers:

$$S(h) \quad = \quad E_h \left\{ \int_0^\infty e^{-rt} \left[I_{h_t > h^s} \;\; \overline{g} + \underline{g} \;\; I_{h_t < h^s} \right] dt \right\} \tag{3}$$

$$\overline{g} \quad = \quad h^s wp \qquad\qquad \underline{g} = -h^s w$$

[6] With fixed workforce, both z and h evolve as a geometric brownian motion, i.e. $dx = x\mu dt + x\sigma dW \quad x = z, h$. Thus, $dx = 0$ if $x = 0$.

In this formula, $I_{()}$ represents an indicator which is equal to one when the attached inequality is true and zero otherwise. $\overline{g}$ and $\underline{g}$ represent instead the pay-off rates from the workforce addition respectively in the overtime and in the undertime region. Adding dL workers *reduces* overtime hours by $h^s\ dL$ and overtime costs by $h^s wp\ dL$ in the overtime region, the pay-off is therefore *positive* $[\overline{g} > 0]$. By contrast, adding dL workers *increases* undertime by $h^s\ dL$ and undertime costs by $h^s w\ dL$ in the undertime region, this obviously contributes *negatively* to the shadow value of workforce $\left[\underline{g} < 0\right]$.

2.3 Shadow Value

In this subsection we solve the expected value in 3 and express $S(h)$ in analytical terms. The jump in the pay-off at h^s, however, rules out the adoption of standard techniques and motivates the development of a method to handle stochastic integration with discontinuous functions. To implement this method we first need to rewrite $S(h)$ as an integral across future h-states. For this purpose, let $P(\widetilde{h}; h, t)$ represent the probability for h_t to be equal or lower than $\widetilde{h}$ given that, at the current time 0, working hours are equal to h:

$$P(\widetilde{h}; h, t) = prob[h_t \leq \widetilde{h} \mid h_0 = h]$$

Using P to substitute the expectation operator in 3, we can rewrite $S(h)$ as follows:(*formal derivation of 4 in the appendix*)

$$S(h) = \underline{g} \int_0^{h^s} v(h, \widetilde{h}) d\widetilde{h} + \overline{g} \int_{h^s}^{\infty} v(h, \widetilde{h}) d\widetilde{h} \tag{4}$$

$$v(h, \widetilde{h}) = \int_0^{\infty} e^{-rt} P'(\widetilde{h}; h, t) dt \tag{5}$$

The expression $v(h, \widetilde{h})$ can be interpreted as a state-contingent discount factor. In fact, $v(h, \widetilde{h})$ represents the value in "state" h of an Arrow-Debreu security that pays one euro per unit of time dt whenever the process hits "state" $\widetilde{h}$. As a consequence, the first integral on the RHS of 4 gives the value in state h of an asset that pays one euro per unit of time dt whenever the process lies in the undertime region $[0, h^s)$. The integral then provides the appropriate metric to assess at the current state h the contribution of a workforce addition in the undertime region. Analogously, the second integral on the RHS of 4 gives the appropriate metric to evaluate the addition in the overtime region.

Since h_t evolves as a geometric brownian motion, P is continuously differentiable with respect to $\widetilde{h}$. Equation 5 relates $v(h, \widetilde{h})$ to $P'(\widetilde{h}; h, t)$, which is the derivative of P with respect to $\widetilde{h}$. Intuitively, the integral in 5 adds over time the probability for working hours to be equal to $\widetilde{h}$ multiplied by the appropriate discount factor.

The upshot of transforming equation 3 in equations 4-5 is that, to compute $S(h)$, one needs first to compute $v(h,\widetilde{h})$. This is done below through lemmas 1 and 2.

Lemma 1

$$P'(R\widetilde{h};Rh,t) = P'(\widetilde{h};h,t)/R \quad \text{for any } R>0 \tag{6}$$

Proof

The two geometric brownian motions $\{h_t\}$ and $\{Rh_t\}$ are *spatially homogeneous*:

$$P(R\widetilde{h};Rh,t) = P(\widetilde{h};h,t)$$

Thus:

$$\begin{aligned} P'(R\widetilde{h};Rh,t) &\equiv \frac{dP(R\widetilde{h};Rh,t)}{d(R\widetilde{h})} = \frac{dP(\widetilde{h};h,t)}{d(R\widetilde{h})} = \\ &= \frac{dP(\widetilde{h};h,t)}{d\widetilde{h}}\frac{d\widetilde{h}}{d(R\widetilde{h})} = P'(\widetilde{h};h,t)/R \end{aligned} \tag{7}$$

This ends the proof of lemma 1.$\diamond$

Corollary

$$v(\widetilde{h},\widetilde{h}) = \frac{v(1,1)}{\widetilde{h}} \tag{8}$$

Equations 5 and 6 imply $v(Rh,R\widetilde{h}) = v(h,\widetilde{h})/R$ and, in particular, $v(R\widetilde{h},R\widetilde{h}) = v(\widetilde{h},\widetilde{h})/R$. Equation 8 obtains from positing $R = 1/\widetilde{h}$.$\diamond$

Lemma 2

Let α_1 and α_2 represent the positive and the negative roots of the equation $\rho(\alpha) = \frac{\sigma^2}{2}\alpha^2 + \left(\mu - \sigma^2/2\right)\alpha - r = 0$, then

$$v(h,\widetilde{h}) = \begin{cases} \frac{v(1,1)}{\widetilde{h}}\left(\frac{h}{\widetilde{h}}\right)^{\alpha_1} & \text{if } h \le \widetilde{h} \\ \frac{v(1,1)}{\widetilde{h}}\left(\frac{h}{\widetilde{h}}\right)^{\alpha_2} & \text{if } h \ge \widetilde{h} \end{cases} \tag{9}$$

$$v(1,1) = \frac{\left(1/\alpha_1 - 1/\alpha_2\right)^{-1}}{r} \tag{10}$$

Proof

Let $T(\widetilde{h})$ represent the time when the process firstly hits state $\widetilde{h}$ given that at current time 0 it lies in state h . Discount factors $v(h,\widetilde{h})$ and $v(\widetilde{h},\widetilde{h})$ are, by definition, connected through the relationship (*formal derivation of 11 in the appendix*)

$$v(h,\widetilde{h}) = v(\widetilde{h},\widetilde{h})\ E_h\left[e^{-rT(\widetilde{h})}\right] \tag{11}$$

A standard result from stochastic calculus (see, for instance, Dixit *et al.*, 1999) is that the expected discount factor $E_h[e^{-rT(\widetilde{h})}]$ is given by $\left(h/\widetilde{h}\right)^{\alpha_1}$ when $h \leq \widetilde{h}$ and by $\left(h/\widetilde{h}\right)^{\alpha 2}$ when $h \geq \widetilde{h}$. Combine this result with 8 and 11 and obtain 9.

The value of $v(1,1)$ in 10 is pinned down by imposing equality between the cumulative value of assets $v(h,\widetilde{h})$ and that of a perpetuity that pays one euro per unit of time dt in all states:

$$\int_{\widetilde{h}\geq 0} v(h,\widetilde{h})d\widetilde{h} = \int_0^{\infty} e^{-rt}dt = 1/r \tag{12}$$

This ends the proof of lemma 2.$\diamond$

Substitute 9 and 10 in 4 and compute $S(h)$:

$$S(h) = \begin{cases} \frac{\underline{g}}{r} - \frac{1}{r}\frac{\alpha_2}{\alpha_1-\alpha_2}\left(\overline{g}-\underline{g}\right)\left(\frac{h}{h^s}\right)^{\alpha_1} & h\in[0,h^s) \\ \\ \frac{\overline{g}}{r} - \frac{1}{r}\frac{\alpha_1}{\alpha_1-\alpha_2}\left(\overline{g}-\underline{g}\right)\left(\frac{h}{h^s}\right)^{\alpha_2} & h\in(h^s,\infty) \end{cases} \tag{13}$$

It is straightforward to show that, irrespective of the discontinuity in the payoff function, $S(h)$ is continuous with a continuous first derivative. The sections of $S(h)$ in the overtime and undertime regions *smooth paste* at h^s in the sense that they both converge to the same value $S(h^s)$ with equal first derivatives. The intuition for continuity and smoothness is as follows. The brownian motion for hours implies that, in proximity of h^s - and even during a very small time interval dt - h_t crosses the level h^s many times so that its actual position, on the right or the left of h^s, is truly immaterial if one has to assess the value of an extra worker added to workforce.

It is also straightforward to see that the slope of $S(h)$ is always positive in both regions. This means that the value of an extra worker increases as production and actual working hours increase. As a consequence, the minimum and the maximum of S are located at the extrema of the h-axis:

$$\lim_{h\to\infty} S(h) = \frac{ph^s w}{r} \qquad\qquad \lim_{h\to 0} S(h) = -\frac{h^s w}{r} \tag{14}$$

The intuition for results in 14 is simple. As current working time approaches infinity, overtime holds with certainty at present and in all future periods, thus the per-period reduction in overtime costs $h^s wp$ is discounted at the "certainty" rate r. Analogously, if $h \to 0$, undertime becomes permanent so that the increase in undertime costs is discounted at rate r.

Results in equation 14 take us back to the parameter restriction 2 stated in subsection 2.2. If current working time is h, adding a single worker gives a benefit $S(h)$ whereas the cost of the addition is H. By contrast, firing a single worker entails a cost F and gives a benefit $-S(h)$. It follows that workforce adjustments do not take place even for asymptotic values of h if

$$\lim_{h\to\infty} S(h) < H \quad \text{ad} \quad \lim_{h\to 0} S(h) > -F$$

The latter replicate condition 2 once one substitutes equation 14.

2.4 Hiring and Firing

Up to this point we have assumed that H and F are so large to discourage workforce adjustments in all demand conditions. In the real world, however, firms recruit new workers if demand is large and dismiss workers if demand is low. In terms of the model, this implies that the underlying cost parameters must satisfy a condition which allows for non-zero workforce adjustments. For this purpose, in the present section we abandon the parameter restriction 2 and adopt the following more realistic alternative:

$$\frac{h^s w}{r} > \max[F, H/p] \tag{15}$$

Under this alterative, firms hire and fire at positive rates but only if the shadow value of an extra worker is respectively greater than H and lower than $-F$. More specifically, due to the positive slope of the shadow value, the optimal policy consists of a lower (h_l) and an upper threshold value (h_u) for h. These thresholds define a segment $[h_l$,$h_u]$ such that the shadow value is lower than adjustment costs if h is positioned in the interior while it equates adjustment costs if h lies on the boundaries. Firing and hiring are operated when h moves respectively beyond the upper and the lower boundary of the segment. In this case, the size of workforce interventions is the minimum required to push working time back onto the boundaries. In fact, any further intervention aimed at an internal point would entail an adjustment cost higher than the benefit.

Since the shadow value of workforce is forward-looking, boundary interventions affect the shadow value not only near the boundaries but also in the interior of the inaction segment. Formally, the shadow value with positive interventions $\widetilde{S}(h)$ obtains from the "free-float" shadow value $S(h)$ by adding two extra terms[7]:

$$\widetilde{S}(h) = S(h) + Ah^{\alpha_1} + Bh^{\alpha 2} \tag{16}$$

The two unknown coefficients A and B and the two policy variables h_l and h_u are determined by the well known value matching and "smooth pasting" conditions (Harrison, 1985; Krugman, 1991):

$$\widetilde{S}(h_u) = H \qquad \widetilde{S}(h_l) = -F \tag{17}$$

[7]Let $\Phi(h)$ represent the expectational component of the shadow value due to boundary interventions: $\Phi(h) \equiv \widetilde{S}(h) - S(h)$. Since this component does not imply any payoff flow, the differential equation that governs its value is $r\Phi = E[d\Phi]$. Standard application of Ito's lemma gives the general solution to the equation: $\Phi(h) = Ah^{\alpha_1} + Bh^{\alpha 2}$.

$$\widetilde{S}'(h_u) = 0 \qquad \widetilde{S}'(h_l) = 0 \tag{18}$$

Equations 17 impose, at the boundaries, the equality between adjustment costs and the shadow value. Equations 18 insure that the shape of $\widetilde{S}(h)$ is consistent with rational expectations[8].

In the remainder of this section we use equations 17 and 18 to compute the optimal hiring and firing boundaries h_l and h_u under a set of baseline parameters. We calibrate the model on a yearly basis, parameters are as follows: $\mu = 0.03$, $\sigma = 0.08$, $w = 1$, $r = 0.12$, $p = 0.3$, $F = 855$, $H = 171$, $h^s = 1710$. The value assigned to yearly standard hours h^s corresponds to 38 hours per week times 45 weeks. The values assigned to parameters μ and σ imply that in normal years output increases by 3% but it can increase by 11% in boom years and by -5% in bad years. The values assigned to parameters w, F and H imply that firing and hiring costs amount respectively to 50% and 10% of the standard annual wage bill $h^s w$, these represent reasonable proportions for countries with average employment protection legislation (OECD, 1994 and 1999). Finally, the rather large value for r is intended to capture a positive rate of risk aversion. With these baseline values the boundaries are $h_l = 1639.4$ and $h_u = 1851.3$. This means that layoffs are operated when undertime reaches 1.6 hours per week while hiring takes place when overtime reaches 3.2 hours per week.

3 Worksharing

When firing costs are not prohibitively large, workers face the risk of being laid off during periods of low production. In these circumstances, however, workers can reduce the number of firing and the chance of being fired by accepting a temporary reduction in the number of employment hours and a proportional reduction in the weekly or monthly wage. In this sense, temporary (or contingent) worksharing involves a more equitable rationing of the limited stock of full time jobs supported by demand conditions. All individuals work for less hours instead of having some of them working full time and some others redundant. In many European countries contingent worksharing is also favoured by publicly administered wage-supplement schemes whereby workers are fully or partially insured from the ensuing loss of income.[9]

From the point of view of the firm, contingent worksharing operates as if the statutory working time were temporarily reduced to accommodate the slack in demand. For this purpose, in this section we assume that the statutory working time is given by $h^s - \delta_t$ with δ_t representing the reduction in standard hours due to worksharing. Furthermore, since worksharing is in place only

[8] For a very transparent discussion on this point see Krugman (1991).

[9] In Germany the scheme is called *Kurzarbeitergeld*, in France *Rémunération Chomage Temporarire*, in Italy *Cassa Integrazione Guadagni*. For details on institutional arrangements of single countries see Missoc (Mutual Information System on Social Protection): http://ec.europa.eu/employment_social/spsi/missoc_en.htm.

Panel a: Firing costs 50% of annual wage			
q	h_l	h_u	Average h
0	1639.4 (36.6)	1851.3 (41.2)	1760.9 (39.13)
0.4	1599.1 (35.5)	1834.7 (40.6)	1736.4 (38.6)
0.8	1428.1 (31.7)	1787.8 (39.7)	1654.9 (36.8)
Panel b: Firing costs 100% of annual wage			
q	h_l	h_u	Average h
0	1600.5 (35.6)	1884.9 (41.9)	1770.4 (39.3)
0.4	1535.0 (34.1)	1859.7 (41.3)	1734.1 (38.5)
0.8	1197.8 (26.6)	1792.8 (39.8)	1620.4 (36.0)

Table 1: The impact of worksharing on the hiring and firing policy. Figures represent annual hours (weekly hours in brackets).

when the slack in demand $(h^s - h_t)$ is positive and since the reduction in standard time is, in the real world, proportional to this slack we assume that h^s is given by the following expression:

$$\delta_t = \begin{cases} q(h^s - h_t) & h_t \leq h^s \\ 0 & h_t \geq h^s \end{cases} \tag{19}$$

Parameter $q \in [0, 1]$ measures the intensity of worksharing, if $q = 0$ worksharing is absent whereas if $q = 1$ worksharing eliminates idle hours altogether.

Once one replaces h^s with $h^s - \delta_t$ as the new standard time the undertime cost parameter $\underline{g}$ changes as follows:

$$\underline{g} = -\ h^s w(1 - q) \tag{20}$$

In fact, adding an extra worker implies, in the undertime region, an increase in idle hours given by h^s minus the portion of these hours which is offset by worksharing, that is qh^s. As a consequence, in the undertime region, an extra worker added to workforce involves a per-period undertime cost corresponding to (the absolute value of) the LHS of equation 20. Clearly, a more intense worksharing reduces (the absolute value of) $\underline{g}$. Since worksharing is not operational in the overtime region, the size of $\overline{g}$ turns out to be unaffected.

In table 1 (panel a) we report computations for h_l, h_u and average working time for different values of the worksharing parameter q while all other parameters are set at baseline values. The average working time in the last column is computed by using the ergodic distribution of h over the corresponding support[10].

As expected, the threshold h_l decreases with the extent of worksharing. Thus, according to the original objective, worksharing makes the firm more reluctant to lay-off during a downturn in the

[10] From Harrison (1985): the ergodic density distribution of a (μ, σ)-geometric brownian motion h subject to impulse control at h_l and h_u is $f(h) = \frac{(k-1)h^{k-2}}{h_u^{k-1} - h_l^{k-1}}$, $k = 2\mu/\sigma^2$, $h \in [h_l, h_u]$.

sense that the firm waits for a deeper slack in demand before taking any action. Less expected, the upper threshold h_u also decreases with worksharing. This means that worksharing makes firms more willing to hire during an upturn. The intuition for this result relates to the forward looking nature of workforce decisions. Firms hire more if they take into account that workers engage in worksharing during future periods of low production.

More formally, one may interpret these results as a consequence of the positive impact of worksharing on the "free float" shadow value of workforce:

$$\frac{dS(h_t)}{dq} = h^s w \cdot E_{h_t} \left\{ \int_t^\infty e^{-r(\tau-t)} I_{(h_\tau < h^s)} d\tau \right\} > 0 \tag{21}$$

The derivative makes it clear that the impact of a worksharing agreement on the shadow value depends upon the current state of production. When current working time h_t is deeply below h^s the expected value on the RHS of 21 is large since the indicator $I_{(h_\tau < h^s)}$ is expected to remain equal to one in the near future. This implies that worksharing is powerful in supporting the value of employed workforce during bad times and, as a consequence, in discouraging layoffs. By contrast, when production is sustained, the indicator $I_{(h_\tau < h^s)}$ is expected to be nil in the near future. However, as the expectation horizon moves further into the future, a positive value for the indicator becomes a more likely occurrence. This means that the sign of the derivative remains positive even if h_t is above h^s. The value of the derivative, however, might be low due to heavier discounting. Thus, worksharing is weak in supporting the shadow value of workforce in good times but it nevertheless puts an incentive towards more recruits.

The impact of worksharing on the willingness to hire and fire translates into higher employment levels. This can be grasped immediately from the last column of the table, which clarifies that worksharing reduces the average long-term working time per individual. In turn, for a given exogenous production flow, the reduction in working time is only possible if, on average, workforce increases.

In countries with strict employment protection, as the ones in continental Europe, firing costs are closer to the whole annual wage bill than to the half (OECD, 1994 and 1999). Thus, in panel b we report computations for firing costs equal to wh^s. As in Bertola and Bentolila (1990), we observe that, for given worksharing, the increase in F moves the h_l boundary downwards and the h_u boundary upward. In addition, we also observe that the enlargement of the boundaries is more pronounced if worksharing becomes more intense. What is the source of this interaction between worksharing and firing costs? We know that with a larger F not only firing but also hiring becomes more costly due to larger expenses from reverting, in the future, a current recruitment decision[11]. Thus, as F increases, the firm widens the $[h_l, h_u]$ interval since it wishes to trade off a lower rate of workforce adjustments with deeper and more prolonged unbalances on the intensive

[11] Indeed, this represents one of the main results in Bertola and Bentolila (1990).

margin. From the perspective of this interpretation it is straightforward to understand the role of worksharing. In fact, with more intense worksharing, the *marginal cost* from widening the $[h_l, h_u]$ interval decreases so that the firm, facing a given increase in F, expands the segment to a larger extent.

In addition to enlarging the corridor, however, with worksharing firing costs also produce a downward shift of the latter. This can be seen from the last column of table 1 which shows that an increase in firing costs leads to a long term substitution of workers with hours which depends on the extent of worksharing. If worksharing is absent, the reaction of the firm to an increase in firing costs leads to an increase in average working time and, as a consequence, to a reduction in the number of workers. By contrast, the substitution operates in the opposite direction if worksharing is in place.

Our conjecture on this result is as follows. We have seen that with higher firing costs the firm enlarges the range of inaction and that worksharing reinforces the enlargement. Due to discounting, however, this reinforcement is not symmetric. In particular, the decrease in *marginal costs* from enlarging the inaction interval is more effective in the undertime than in the overtime region. It follows that, with worksharing, the downward shift of h_l due to an increase in F is more pronounced with respect to the upward shift of h_u (see table 1). This implies that the corridor, while enlarging, also "shifts downwards" with the obvious consequence that average working time tends to decrease.

In the theoretical literature, the employment impact of firing restrictions depends on a number of factors and, as a consequence, turns out to be inherently ambiguous. Ljungqvist (2004), for instance, by means of a general equilibrium model with frictional unemployment shows that the impact depends on the details of the stochastic process that drives the productivity of firms. By contrast, Bertola (1992) emphasizes the role of discountig and of decreasing marginal returns in partial equilibrium.

This model contributes to this line of research by introducing into the picture the hours-worker substitutability. More specifically, the paper illustrates that hours-workers substitutability represents a potentially though neglected determinant of the overall employment impact of firing costs. From this perspective, worksharing is relevant in that it affects the terms on which hours and workers are traded-off in the long run.

To grasp further insights on this interaction, in table 2 we compute the impact on the employment policy due to an increase in firing costs and under a symmetric adaptation of statutory hours to business conditions. In other words, table 2 reproduces computations of table 1 under the assumption that, in contrast with real world institutions, standard working time increases with high demand and decreases with low demand:

$$\delta_t = \begin{cases} q(h^s - h_t) & h_t \le h^s \\ q(h_t - h^s) & h_t \ge h^s \end{cases}$$

Panel a: Firing costs 50% of annual wage			
q	h_l	h_u	Average h
0	1639.4 (36.6)	1851.3 (41.2)	1760.9 (39.13)
0.4	1609.4 (35.8)	1882.6 (41.8)	1771.6 (39.4)
0.8	1468.0 (32.6)	1961.6 (43.6)	1795.1 (39.9)
Panel b: Firing costs 100% of annual wage			
q	h_l	h_u	Average h
0	1600.5 (35.6)	1884.9 (41.9)	1770.4 (39.3)
0.4	1546.8 (34.4)	1916.3 (42.6)	1777.7 (39.5)
0.8	1233.3 (27.4)	1976.8 (43.9)	1778.9 (39.5)

Table 2: The employment policy under a symmetric adaptation of standard hours.

It is clear from the table that a symmetric adaptation of standard hours to demand does not involve any meaningful change in average working time and, as a consequence, in average employment for given firing costs. In addition, an increase in firing costs widens the corridor but leaves average working time unaffected for any level of q.

4 Conclusions

We have studied the effects of contingent worksharing in a stochastic cost-minimisation setting where firms are allowed to change the labour input along the extensive as well as the intensive margin. In this context, contingent worksharing benefits workers retention in bad times and increases the number of recruits in good times. The upshot of these two effects is a reduction in the long run average working time and, as a consequence, an increase in the employment level for given long run average production.

We have also studied the role of worksharing in determining the impact of firing cost on the optimal employment policy. First, firing costs discourage workforce adjustments to an extent which depends on worksharing. Second, worksharing reinforces the impact of firing costs on the firing boundary more than that on the hiring boundary with the consequence that long run working time decreases. Thus, worksharing bends the impact of higher firing costs towards an increase in the employment level.

From an empirical perspective, these results call for a proper account of the interaction between employment protection and measures of worksharing incidence in unemployment regressions. To our knowledge, this has not yet been done in the large applied macro-labour literature.

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APPENDIX

This appendix is only intended to ease the process of refereeing.

From equation 3 to equations 4-5.

$$
\begin{aligned}
S(h) &= E_h \left\{ \int_0^\infty e^{-rt} \left[I_{h_t > h^s} \;\; \overline{g} + \underline{g} \;\; I_{h_t < h^s} \right] dt \right\} = \\
&= \overline{g} \left\{ \int_0^\infty e^{-rt} E_h \left(I_{h_t > h^s} \right) dt \right\} + \underline{g} \left\{ \int_0^\infty e^{-rt} E_h \left(I_{h_t < h^s} \right) dt \right\} = \\
&= \overline{g} \left\{ \int_0^\infty e^{-rt} \left[1 - E_h \left(I_{h_t \leq h^s} \right) \right] dt \right\} + \underline{g} \left\{ \int_0^\infty e^{-rt} E_h \left(I_{h_t < h^s} \right) dt \right\} = \\
&= \overline{g} \left\{ \int_0^\infty e^{-rt} \left[1 - P\left(h^s; h, t \right) \right] dt \right\} + \underline{g} \lim_{x \to (h^s)^-} \left\{ \int_0^\infty e^{-rt} P\left(x; h, t \right) dt \right\} = \\
\text{(note 1)} &= \overline{g} \left\{ \int_0^\infty e^{-rt} \left[1 - P\left(h^s; h, t \right) \right] dt \right\} + \underline{g} \left\{ \int_0^\infty e^{-rt} P\left(h^s; h, t \right) dt \right\} = \\
&= \overline{g} \left\{ \int_0^\infty e^{-rt} \left[\int_{h^s}^\infty P'\left(\widetilde{h}; h, t \right) d\widetilde{h} \right] dt \right\} + \underline{g} \left\{ \int_0^\infty e^{-rt} \left[\int_0^{h^s} P'\left(\widetilde{h}; h, t \right) d\widetilde{h} \right] dt \right\} = \\
&= \overline{g} \left\{ \int_{h^s}^\infty \left[\int_0^\infty e^{-rt} P'\left(\widetilde{h}; h, t \right) dt \right] d\widetilde{h} \right\} + \underline{g} \left\{ \int_0^{h^s} \left[\int_0^\infty e^{-rt} P'\left(\widetilde{h}; h, t \right) dt \right] d\widetilde{h} \right\} = \\
\text{(note 2)} &= \overline{g} \left\{ \int_{h^s}^\infty v(\widetilde{h}, h) d\widetilde{h} \right\} + \underline{g} \left\{ \int_0^{h^s} v(\widetilde{h}, h) d\widetilde{h} \right\}
\end{aligned}
$$

Note 1: Since $P\left(x; h, t\right)$ is differentiable with respect to x it is also left-continuous.

Step 2: Substitute equation 5

Equation 11

Equation 11 is intuitive and holds by definition. Nevertheless one can derive the equation from the definition of $v(h,\widetilde{h})$ in 5:

$$
\begin{aligned}
v(h,\widetilde{h}) &= \int_0^\infty e^{-rt} P'(\widetilde{h};h,t)dt = \\
&= E_h\left[\int_0^\infty e^{-rt} P'(\widetilde{h};h,t)dt\right] = \\
&= E_h\left[\int_0^{T(\widetilde{h})} e^{-rt} P'(\widetilde{h};h,t)dt + \int_{T(\widetilde{h})}^\infty e^{-rt} P'(\widetilde{h};h,t)dt\right] = \\
&= E_h\left[\int_0^{T(\widetilde{h})} e^{-rt} P'(\widetilde{h};h,t)dt\right] + E_h\left[e^{-rT(\widetilde{h})}\int_{T(\widetilde{h})}^\infty e^{-r[t-T(\widetilde{h})]} P'(\widetilde{h};h,t)dt\right] = \\
\text{(note 1)} &= E_h\left[e^{-rT(\widetilde{h})}\int_{T(\widetilde{h})}^\infty e^{-r[t-T(\widetilde{h})]} P'(\widetilde{h};h,t)dt\right] = \\
\text{(note 2)} &= E_h\left[e^{-rT(\widetilde{h})}\right] E_h\left[\int_{T(\widetilde{h})}^\infty e^{-r[t-T(\widetilde{h})]} P'(\widetilde{h};h,t)dt\right] = \\
\text{(note 3)} &= E_h\left[e^{-rT(\widetilde{h})}\right] E_h\left[\int_0^\infty e^{-r\tau} P'\left(\widetilde{h};h,\tau+T(\widetilde{h})\right)d\tau\right] = \\
\text{(note 4)} &= E_h\left[e^{-rT(\widetilde{h})}\right] E_h\left[\int_0^\infty e^{-r\tau} P'(\widetilde{h};\widetilde{h},\tau)d\tau\right] = \\
&= E_h\left[e^{-rT(\widetilde{h})}\right] E_h\left[v(\widetilde{h},\widetilde{h})\right] = \\
&= E_h\left[e^{-rT(\widetilde{h})}\right] v(\widetilde{h},\widetilde{h})
\end{aligned}
$$

Note 1: $P'(\widetilde{h};h,t) = 0$ if $t < T(\widetilde{h})$

Note 2: Factors are independent by the Strong Markov Property

Note 3: Change of variable: $\tau = t - T(\widetilde{h})$

Note 4: $P'\left(\widetilde{h};h,\tau+T(\widetilde{h})\right) = P'(\widetilde{h};\widetilde{h},\tau)$, see below:

$$P'\left(\widetilde{h};h,\tau+T(\widetilde{h})\right)d\widetilde{h} = \int_{y\in(0,\infty)} P'\left(y;h,T(\widetilde{h})\right)\cdot P'(\widetilde{h};y,\tau)dy = P'(\widetilde{h};\widetilde{h},\tau)d\widetilde{h}$$

This is true since

$$P'\left(y;h,T(\widetilde{h})\right) = \begin{cases} 0 \text{ if } y \neq \widetilde{h} \\ 1 \text{ if } y = \widetilde{h} \end{cases}$$

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