UNIVERSITA’ CATTOLICA DEL SACRO CUORE
- Milano -

QUADERNI DELL’ISTITUTO DI
ECONOMIA DELL’IMPRESA E DEL LAVORO

Unions, Job Protection and Employment

Giulio Piccirilli

n. 40 – marzo 2005
Unions, Job Protection and Employment

Giulio Piccirilli*

Abstract

In this paper we study a dynamic interaction between a single wage-setting union and a mass of small competitive firms. Firms are subject to stochastic changes in their economic environment and to costly labour shedding. The game is solved under the assumption that the union commits to a given wage sequence as well as under the assumption that such a commitment is not feasible.

We find that firing costs are nearly neutral for the level of employment both in the commitment and in the no-commitment equilibrium if the objective function of the union is linear with respect to the wage rate. By contrast, if the function is concave, firing costs decrease employment in the no-commitment equilibrium while neutrality survives under commitment.

We argue that these findings shed some light on the robustness of predictions obtained by models of dynamic labour demand as well as by those explanations of unemployment built around the insider-outsider mechanism.

JEL-Code: J23, J51, J63

Keywords: Employment, Union, Firing Costs.

*Istituto di Economia dell’Impresa e del Lavoro (IEIL), Università Cattolica, Largo Gemelli 1, 20123 Milan. Mail: giulio.piccirilli@unicatt.it
1 Introduction

The effects of mandated job protection have received great attention by the economic profession over the last fifteen years. There is still a lack of consensus, however, regarding the employment effect of firing costs. According to models of dynamic labour demand, firing costs reduce workforce turnover with no significant effects on average employment (Bentolila and Bertola, 1990). By contrast, the insider-outsider theory explains that firing costs reduce employment by contributing to the bargaining power of insiders and, henceforth, by supporting high wage claims (Lindbeck and Snower, 1988).

An obvious way to ascertain which of the two views is closer to the real world is that of comparing the labour market performance of countries with different regimes of employment protection. Empirical research, however, has up to now failed to provide a definitive answer on the issue. Lazear (1990) and, more recently, Djankov et al. (2003) are just two examples of empirical works which find that dismissal regulations increase unemployment. By contrast, Bertola (1990), the OECD (1999) and several others find that aggregate employment levels are not affected by the stringency of legal provisions.

In retrospect, the conflict between predictions is a result of differences in models core assumptions. First, in contrast with the insider-outsider theory, models of dynamic labour demand assume wages to be exogenous and, henceforth, rule out any effect of firing costs which operate through the wage-setting mechanism. Second, in contrast with models of dynamic labour demand, the insider-outsider theory is developed within a static economic environment and, henceforth, is deprived of any predictive power regarding the average employment level in an intrinsically dynamic context.

In this paper we offer new theoretical insights on the issue by using a model which removes above special assumptions. On the one hand, in our setting, business conditions change randomly from time to time so that firms and workers are compelled to make decisions in a dynamic stochastic environment. On the other hand, wages are set by a forward looking union instead of being exogenously given. In this respect, we study both the case where the union can commit to future wages and the case where such a commitment is not feasible.
The outcomes of our analysis are the following. First, we find that the equilibrium under commitment presents results that are qualitatively similar to those of models of dynamic labour demand. Firing costs reduce workforce turnover when business conditions change while average employment is hardly and ambiguously affected. Thus, predictions from this class of models appear robust to the introduction of endogenous wages provided one assumes wage predetermination. Second, the no-commitment equilibrium presents results that are reminiscent of those from the insider-outsider theory. The union increases the wage by the full amount of firing costs after new workers have been hired in a business upsurge. In turn, firms anticipate the wage increase and exhibit reluctance to hiring. However, in contrast with the insider-outsider theory, lower employment levels do not necessarily follow. This happens because the union tries to counteract firms reluctance by charging particularly low wages at the time of hiring. More specifically, we find that wages at the time of hiring may be set at a level so low that the no-commitment equilibrium exhibits the same level of employment than the equilibrium under commitment. This happens if the union is utilitarian and if the utility function of workers is linear with respect to the wage rate. Thus, predictions from the insider-outsider theory appear robust to a stochastic dynamic extension of the original model provided one rules out wage predetermination. In addition, a sufficient curvature of the objective function of individuals represents a further necessary conditions for the insider-outsider mechanism to be effective.

Summing up, we show that the conflicting views in the existing literature may be both correct depending on the institutional setting which surrounds the wage bargaining process. By doing so, we implicitly suggest empirical investigators to pay more attention to the interaction between variables that capture the bargaining context and measures of employment protection in cross-country analyses. Corporatism, in fact, may favour commitment-like equilibria while a non-cooperative environment may favour no-commitment equilibria. Failing to account for this interaction is bound to bias the coefficient that captures the effect of employment protection on the level on unemployment. We regard these conclusions as the main contribution of the paper.

Works focusing on firms-union strategic interactions in presence of adjustment
costs are rare. The closest to the present paper are Kennan (1988), Lockwood and Manning (1989) and Modesto and Thomas (2001). All these papers, however, deal with different issues and, with the exception of Kennan, use a deterministic setting. The first two works investigate whether the speed of adjustment of employment towards its long run equilibrium is slower in a unionised market as opposed to a competitive one. Modesto and Thomas, instead, investigate if the speed of adjustment depends on whether the union is able to make a wage commitment.

The concern on the dynamics of the adjustment path is closely related to the way adjustment costs are modeled. All these papers, in fact, adopt quadratic symmetric costs and, as a consequence, find that workforce changes are spread over long periods of time. Quadratic costs, however, do not square with legal provisions (Nickell, 1986). Further, continuous small variations stand in sharp contrast with empirical analyses conducted on firm level data which document that the dominant pattern is made of sporadic discrete adjustments followed by long spells of inaction (Hamernesh, 1989; Caballero et al., 1997). Symmetry appears also problematic as the main component of adjustment costs is very likely to be related to workforce dismissals instead of additions (Emerson, 1988).

Finally, Modesto and Thomas also study whether the ability to commit affects wages and employment and reach conclusions that are close to ours. In their framework, however, the shape of workers preferences has no role while quadratic costs are essential for their results to hold. For these reasons, we regard our paper as complementary to theirs.

The plan of the paper is as follows. In section 2 we present the economic environment. In section 3 and 4 we study the firms-union interaction respectively with and without a commitment on wages and under a fairly general union objective function. In section 5 we compare the two equilibria under the assumption that the union is utilitarian and establish under what conditions they are equivalent. Section 6 contains some concluding remarks.
2 The economic environment

A single wage-setting union and a unit mass of identical competitive firms operate in the same industry. Business conditions, i.e. demand and productivity conditions, are common to all firms and are subject to stochastic changes. Firms maximise the discounted cash flow by adopting an optimal employment policy. In making their decisions, they are constrained by the obligation to pay to a third party a firing cost for any dismissed worker.\footnote{A per-person hiring cost could also be fitted into the model with no relevant changes. We decided to abstract from hiring costs to keep the model as simple as possible. Further, in the real world, mandated hiring costs are much lower in size with respect to firing costs (Emerson, 1988).} Production is realised through a labour-only technology, the current cash flow $f_t$ for the representative firm is given by the difference between current revenues and labour costs. Finally, we assume that the marginal revenue is a decreasing linear function of the employment level:

$$f_t = (\alpha_t - \frac{d}{2}l_t)l_t - w_t l_t - I_{l_t \leq l_{t-1}} F (l_{t-1} - l_t)$$

Revenues $(\alpha_t - \frac{d}{2}l_t)l_t$ depend on the level of firm’s employment $l_t$ and on the shifter $\alpha_t$ which indexes business conditions during period $t$. The value of the shifter may change from period $t$ to period $t+1$. We assume that the motion of $\alpha$ is governed by a two states Markov process, $\alpha$ cycles between an high value $\alpha_g$ and a low value $\alpha_b$ ($< \alpha_g$) with a constant per-period transition probability $q(< 1)$. Labour costs are given by the wage bill $w_t l_t$ plus total firing costs. $F$ represents the firing cost for a single dismissed worker while $I_{l_t \leq l_{t-1}} (l_{t-1} - l_t)$ gives the total number of dismissed workers. The indicator $I_{l_t \leq l_{t-1}}$ switches from 1 to 0 if current employment becomes strictly higher than past employment.

The union maximizes a discounted utility flow by adopting an optimal wage policy (monopoly union). The per-period union objective function $U(w_t, L_t)$ depends on the current wage rate $w_t$ and the current aggregate employment $L_t$. This function is characterised by the following properties:
Assumptions on the shape of $U$ and of the revenue function guarantee that both goods, wages and employment, are normal. In the static textbook monopoly model, under these assumptions, the union chooses higher wage and employment levels if labour demand moves upward.

3 Wages and employment under commitment

3.1 The optimal hiring and firing policy

In this section, we study the firms-union interaction under the assumption that the union announces a particular wage sequence at the beginning of the game and makes a binding commitment to it.

The motion of the forcing variable $\alpha$ leads to a stochastic business cycle at industry level. Spells of good business conditions alternate with spells of bad conditions, the duration of each spell is random. For notational convenience, we order business spells according to their timing and use the index $n \in \mathcal{N}$ to express the ordering. Thus, the first spell ($n = 1$) is the one which begins at the outset of the game. The second spell ($n = 2$) starts after the first, at the time initial business conditions give way to new business conditions. The third and all other following spells are defined in the same manner.

We assume that initial business conditions are good and that firms starts with zero employment. Knowing business conditions in the first spell is sufficient to establish what business conditions characterise the $n$-th spell. Equivalently, one may think of a function $\alpha_n : \mathcal{N} \to \{\alpha_g, \alpha_b\}$ which for any odd $n$ picks up the value $\alpha_g$ and for any even $n$ the value $\alpha_b$ (figure 1).

The union announces its policy at the beginning of the first spell. At this time it commits to a wage sequence of the type $\{w_{n,\tau}\}$ with $n = 1, 2, 3... \tau = 1, 2...$ This
sequence implies that the wage is made contingent on the two state variables that evolve exogenously. Less formally, the wage is allowed to change along any given business spell as well as across different spells for any given elapsed duration $\tau$. Given the wage sequence $\{w_{n,}\}$, the optimal employment sequence or, equivalently, the optimal hiring and firing sequence, solves the following Bellman problem:

$$V_{n,\tau}(l-1) = \max_{l'} \left( \alpha_n - \frac{d}{2}l' - w_{n,\tau}l' - I_{l' \leq l-1} F(l-1 - l') + \right.$$

$$+ \frac{1}{1+r} \left[ qV_{n+1,1}(l') + (1-q)V_{n,\tau+1}(l') \right]$$

$V_{n,\tau}(l-1)$ represents the value of the firm, which depends on the level of employment inherited from the past $l-1$ and on the state vector $(n, \tau)$. The value of the firm is given by the sum of the current cash flow plus the expected discounted continuation value. Notice that business conditions in the next period either change (with probability $q$) or remain constant (with probability $1-q$). In the first case the state vector becomes $(n+1, 1)$ since the $(n+1)$-th spell begins. In the second case the vector becomes $(n, \tau + 1)$ as the only state variable that changes is the elapsed duration of the current spell.

To characterise the optimal hiring and firing policy in intuitive terms we introduce the notion of the shadow value of labour. We define the shadow value $S_{n,\tau}(l'; l-1)$ as the variation in the current value of the firm $V_{n,\tau}(l-1)$ following a marginal upward shift in the employment path $\{l-1, l',.....\}$. The shift is computed along the optimal hiring and firing policy so that, by the envelope theorem,

\[2\] Of course, lagged employment represents a third endogenous state variable. Commitment strategies conditional on exogenous state variables are often referred as "open loop strategies".
$S_{n,\tau}(l'\mid L_{-1})$ also coincides with the derivative of $V_{n,\tau}(l_{-1})$ with respect to $l_{-1}$.

Thus, we differentiate $V_{n,\tau}(l_{-1})$ by taking account that a marginal increase in $l_{-1}$ is accompanied by an equal increase in $l'$ and express $S_{n,\tau}(l'\mid L_{-1})$ in recursive form:

$$S_{n,\tau}(l'\mid L_{-1}) = \alpha_n - dl' - w_{n,\tau} + \frac{1}{1+r} [qS_{n+1,1}(\cdot;l') + (1-q)S_{n,\tau+1}(\cdot;l')] \quad (2)$$

The current shadow value is given by the current net marginal revenue of labour plus the expected discounted next period shadow value. Equation 2 can be interpreted as a labour demand schedule given in implicit form. In fact, if the whole sequence of shadow values were given, the equation would specify the slope of the schedule as well as its position in the wage-employment space $[l', w_{n,\tau}]$. The sequence of shadow values, however, is itself part of the equilibrium as it depends on the future sequence of employment and wage levels (see equation 7 below). Accordingly, the position of labour demand for any state of the game is part of the equilibrium path. Finally, equation 2 makes clear that, due to the fact that adjustment costs are linear, $S_{n,\tau}(l'\mid L_{-1})$ depends on lagged employment $L_{-1}$ only through the effect of the latter on the choice of $l'$.

We are now ready to characterise the optimal employment policy. Observe that the derivative of $V_{n,\tau}(l_{-1})$ with respect to $l'$ jumps from $S_{n,\tau}(l'\mid L_{-1})$ to $S_{n,\tau}(l'\mid L_{-1}) + F$ depending on whether $l'$ moves in the region above or below $L_{-1}$. This implies that $S_{n,\tau}(l'\mid L_{-1})$ represents the gain from hiring an extra unit of labour while $-S_{n,\tau}(l'\mid L_{-1}) + F$ the gain from firing. As usual when dealing with discontinuous derivatives, inaction may turn out to be the optimal decision. In the present case, inaction means that current employment does not change from its lagged level. This happens when both types of action, hiring and firing, entail a negative return or, more formally, when the shadow value under inaction $S_{n,\tau}(L_{-1}\mid L_{-1})$ is negative but greater than $-F$.

Positive workforce adjustments occur only when the shadow value under inaction falls outside the interval $[-F, 0]$. If $S_{n,\tau}(L_{-1}\mid L_{-1})$ is positive, optimality requires recruiting new workers. Further, hiring must take place up to the point the marginal recruit becomes valueless or, more formally, up to the point the shadow value is reset to the upper boundary of the interval (hiring boundary). By contrast, when
$S_{n,\tau}(l_{-1}; l_{-1})$ is lower than $-F$, firms ought to fire until the shadow value is reset to the lower boundary of the interval (firing boundary).\(^3\)

This policy is summarised by the following conditions:

\begin{align*}
S_{n,\tau}(l'; l_{-1}) &= 0 \quad \text{if} \quad l' > l_{-1} \\
S_{n,\tau}(l'; l_{-1}) &= -F \quad \text{if} \quad l' < l_{-1} \\
\quad l' &= l_{-1} \quad \text{if} \quad 0 \leq S_{n,\tau}(l_{-1}; l_{-1}) \leq -F
\end{align*}

These conditions suggest that the choice of $l'$ depends, beyond the wage sequence, also on which of the three cases above holds and, ultimately, on the size of lagged employment $l_{-1}$.

In turn, the link between current and past employment implies that, for the union, the level of employment from the first period of the second spell onwards is a stochastic variable. This is due to the fact that the union does not know with certainty when the first spell terminates and, henceforth, it does not know with certainty what is the level of lagged employment at the beginning of the second spell. Uncertainty, however, is not present if equilibrium employment is constant along any business spell. In this case, in fact, the level of employment in the last period of any spell is independent from the stochastic duration of the spell.

In the next subsection, we prove that a feature of the equilibrium path is constant employment within spells (proposition 1). This allows us to continue our analysis in this section under the conjecture that $l_{n,\tau}$, the level of employment in the $\tau$-th period of the $n$-th spell, is perfectly known by the union and the firms at the outset of the game.

For future reference, we end this sub-section by presenting a different formula for $S_{n,\tau}$. If one runs forward equation 2, $S_{n,\tau}$ can be expressed as the expected discounted sum of net marginal revenues in all future periods. To write such an expression, however, we need to bring some more structure into the model. Thus,

\(^3\)The shadow value is clearly decreasing with respect to the current employment level $l'$, see equation 7 below.
first we introduce the function $p[(n, \tau); (n', \tau')]$, which gives the transition probability from the current state $(n, \tau)$ to state $(n', \tau')$ after an interval of exactly $s$ periods. Second, we define with $T[(n, \tau), (n', \tau')]$ the value in state $(n, \tau)$ of an asset that pays one euro when state $(n', \tau')$ occurs. Using transition probabilities $p$, $T$ is defined as follows:

$$
T[(n, \tau), (n', \tau')] = \sum_{s=0}^{\infty} p[(n, \tau); (n', \tau'), s] \left( \frac{1}{1 + r} \right)^s \quad n' \geq n
$$

(6)

Accordingly, the required formula for the shadow value $S_{n, \tau}$ is

$$
S_{n, \tau} = \sum_{g=\tau}^{\infty} T[(n, \tau), (n, g)] \{\alpha_n - dl_{n,g} - w_{n,g}\} +
$$

$$
+ \sum_{m=n+1}^{\infty} \sum_{g=1}^{\infty} T[(n, \tau), (m, g)] \{\alpha_m - dl_{m,g} - w_{m,g}\}
$$

(7)

The first line refers to the stream of future net marginal revenues along the current $n$-th spell; the second line refers to the stream along future spells.

### 3.2 The optimal wage policy

The union chooses the sequence $\{w_{n,\tau}\}$ with the objective of maximising the discounted flow of utility

$$
W = \sum_{n=1}^{\infty} \sum_{\tau=1}^{\infty} T[(1, 1), (n, \tau)] U(w_{n,\tau}; L_{n,\tau})
$$

(8)

subject to the optimal employment policy of firms which constrains the shadow value within the closed interval $[-F, 0]$:

$$
S_{n, \tau} \leq 0 \quad -F - S_{n, \tau} \leq 0 \quad \forall n, \tau
$$

(9)

$S_{n, \tau}$ given by equation 7

---

4Of course, if state $(n', \tau')$ has already occurred $[n' < n$, for instance] the value of $T$ is nil. Borrowing from the general equilibrium theory, $T$ can be regarded as a pricing function for state-contingent Arrow-Debreu securities.
In the appendix, we study this problem and prove that combining the f.o.c.s for \( w_{n,\tau} \) and \( l_{n,\tau} \) one obtains that, along the equilibrium path, the marginal rate of substitution between employment and wages is equated in all states to the slope of marginal productivity:

\[
U_t(w_{n,\tau}, l_{n,\tau}) = dU_w(w_{n,\tau}, l_{n,\tau}) \quad (10)
\]

In the appendix, we also prove the following proposition:

**Proposition 1**

*In the commitment equilibrium,*

a) the wage and the employment level are constant within any spell;

b) the shadow value of labour is constant within any spell.

**Proof:** see Appendix.

The proof of part a) hinges on the normality of employment and wages (equation 1) as well as on the forward looking nature of labour demand (equation 2). Normality, coupled with the tangency condition 10, implies that employment and wages increase from the current period \( \tau \) to period \( \tau + 1 \) only if labour demand moves up. However, within spells business conditions are constant by definition so that labour demand moves up from \( \tau \) to \( \tau + 1 \) only if at time \( \tau \) demand is low due to the expectation of firings at \( \tau + 1 \). Summing up, one can have hirings at \( \tau + 1 \) with unchanged business conditions only if firms anticipate firings, a contradiction. An analogous argument can be made to rule out within-spell firing.

The proof of part b) uses the following argument. Constant employment and wage levels coupled with constant business conditions imply that the unique non-explosive \( S \)-path that solves the difference equation 2 is a constant \( S \). Explosive \( S \)-paths are not possible as they would violate in finite time the \([-F, 0]\) restriction imposed by the optimal employment policy of firms.

The combination of the tangency condition 10 and proposition 1 imply that, depending on parameters, only two equilibrium paths may arise. The first is an equilibrium where hiring takes place only at the outset of the game whereas inaction follows thereupon. In this equilibrium the shadow value is zero along good spells
and negative but higher than $-F$ along bad spells. The second is an equilibrium where hiring takes place at the beginning of good spells and firing at the beginning of bad spell. We regard this equilibrium as the one which is more likely from an empirical point of view. Thus, in the remainder of this section we compute wage and employment levels along such an equilibrium and give a necessary and sufficient condition for its existence in the form of a restriction on parameters.

Let the couple $L_{g,c}, w_{g,c}$ [c: commitment] represent the aggregate employment and the wage solution in good spells and $L_{b,c}, w_{b,c}$ the corresponding solution in bad spells. Positive adjustments and proposition 1 (part b) imply that the shadow value is zero along good spells and $-F$ along bad spells. Inserting these values in the implicit labour demand and coupling the latter with equation 10 we end up with two systems that solve for the levels of employment and wages in the two states:

$$U_l(w_{g,c},L_{g,c}) = d U_w(w_{g,c},L_{g,c})$$

$$w_{g,c} = -q \frac{F}{1+r} + \alpha_g - dL_{g,c}$$

$$U_w(w_{b,c},L_{b,c}) = d U_l(w_{b,c},L_{b,c})$$

$$w_{b,c} = r + q \frac{F}{1+r} + \alpha_b - dL_{b,c}$$

Thus, at any time along the equilibrium path the union charges the ”static” monopolistic wage, corresponding to the tangency between labour demand and the highest indifference curve. The dynamic equilibrium takes the form of simple collection of purely static equilibria. In contrast with the static case, however, the position of labour demand in the wage-employment space is endogenous.

We conclude this section by stating a proposition which identifies a necessary and sufficient condition for having an equilibrium with positive adjustments.

\footnote{Notice that in equations 12 and 14 we have used aggregate employment in substitution of firm level employment. Formally, we have applied the rule whereby individual and aggregate variables are the same in a context featuring a unit mass of homogeneous agents.}
Proposition 2

If the inequality

\[ \alpha_g - \alpha_b > \frac{r + 2qF}{1 + r} \]  \hspace{1cm} (15)

holds, then in the commitment equilibrium firms hire at the beginning of good spells and fire at the beginning of bad spells.

Proof

For positive hiring at the beginning of good spells and positive firing at the beginning of bad spells it must be true that \( L_{g,c} > L_{b,c} \). Since the properties that characterise the union objective function imply that employment and wages are normal goods, this amounts to require that labour demand in good times lies above labour demand in bad times in the wage-employment space. That is, by inspecting demand schedules 12 and 14, it amounts to impose the restriction in equation 15.

In the remainder of this section we hold the restriction in equation 15 to be true. Intuitively, positive adjustments arise if firing costs are sufficiently low and/or the change in business conditions sufficiently large. Further, notice that firing costs enter the inequality in combination with the transition rate \( q \). An higher transition probability makes business spells less durable and subtract incentives to workforce adjustments. For this reason, for given firing costs, the inequality tends to be true for low values of \( q \).

3.3 Workforce stability and employment

Labour demand schedules 12 and 14 convey the main result of models of dynamic labour demand. In fact, these schedules are similar to those usually derived in a context of exogenous wages (Bertola, 1990). Here, we have shown that they hold also if wages are set by a monopoly union provided the latter commits to the whole sequence of future wages.

Observe that firing costs insert a wedge between the wage and the marginal labour revenue. The sign of the wedge is positive during bad spells and negative during good spells. In graphical terms, this amounts to say that firing costs
shift labour demand up in bad times and down in good times. The first effect is straightforward, the second is due to the expectation of a future reversal in business conditions. In turn, since employment and wages are normal goods for the union, a lower labour demand in good times leads to lower levels of both variables. By contrast, an higher labour demand in bad times leads to higher employment and wage levels. The upshot of these effects is that firing costs tend to compress fluctuations in wage and employment levels that take place at business turns.

Smaller employment and wage fluctuations, however, are not accompanied by significant and unambiguous changes in their average level. These changes, in fact, depend on a "discounting effect" (regulated by \( r \)) and on a "curvature effect" which relates to the shape of union indifference curves (regulated by \( U_t, U_w \)).

Discounting makes firing costs more relevant for firing decisions than for hiring decisions. Formally, the wedge in equation 12 is smaller in absolute size than the one in equation 14 by an amount which increases with respect to \( r \). As a consequence, the upward shift of the schedule in bad times is more pronounced in comparison to the downward shift in good times. This effect - taken alone - obviously leads to an increase in the average employment and wage levels.

Notice finally that, for a given interest rate, the "discounting effect" decreases with respect to the state reversion probability \( q \). In a highly volatile environment (high \( q \)) the chance of being compelled to fire in a short while is large and, as a consequence, firing costs have a substantial effect also on the hiring margin.

In contrast with the "discounting effect", the "curvature effect" is not clear-cut. In a stochastic cycle of low and high demand schedules the curvature of indifference curves clearly matters. What is relevant in the present context is that, in the absence of the "discounting effect" (\( r = 0 \)), the "curvature effect" could be of any sign depending on the shape of the union utility function. If indifference curves are homothetic, for instance, this effect can be shown to be nil. An increase in firing costs reduces workforce turnover but does not affect average employment. With different preferences, however, an increase in firing costs may lead to higher as well as lower levels in average employment.\(^6\)

\(^6\)If the objective function is, for instance, \( U(w, L) = L[\log(w) - \log(\bar{w})] \) an increase in firing costs reduces average employment whereas the function \( U(w, L) = w[\log(L) - \log(\bar{L})] \) leads to the
4 Wages and employment without commitment

In this section we analyse the firms-union interaction in the absence of a wage commitment. We assume that all agents play a game where decisions are optimal at any point in time conditional on the current state of the game, on other players’ current actions and on the expected future actions by all players. Since the dynamics is governed by a Markov process, if each player conjectures that all other players adopt Markov strategies, i.e. their actions are made contingent only on the current state, the current state also encapsulates all relevant information for expectations held at current time. This implies that optimal decisions depend ultimately only on the current state and, as a consequence, that conjectures are self-fulfilling.

Let the current state from the point of view of the union be summarised by the vector \((j, L_{t-1})\) with \(j\) \((j = g, b)\) representing business conditions at current time. The optimal wage policy takes the form of a function of the state vector: \(w(j, L_{t-1})\). From the point of view of each single firm, due to the presence of adjustment costs, the state of the game also includes its own level of lagged employment so it should be represented by the triple \((j, l_{t-1}, L_{t-1})\). Observe, however, that the union strategy implies that, for given business conditions, the wage is a sufficient statistics for lagged aggregate employment \(L_{t-1}\). Thus, the state of the game for each single firm can be equivalently represented by the triple \((j, l_{t-1}, w_t)\). The ensuing optimal employment strategy is of the form \(l(j, l_{t-1}, w_t)\).

Equilibrium arises when wage and employment strategies are mutual best responses. This notion is often referred as (subgame perfect) Markov equilibrium, (Maskin e Tirole, 1988).

4.1 The employment strategy

Let us indicate with \(S[j, l_{t-1}, w_t]\) the shadow value of labour in the Markov equilibrium:
\[ S[j, l_{t-1}, w_t] = \alpha_j - d l - w_t + \]
\[ + \frac{1}{1 + r} [qS[j-1, l, w(j-1, L)] + (1 - q)S[j, l, w(j, L)]] \]

\[ S[j, l_{t-1}, w_t] = \alpha_j - d l - w_t + \]
\[ + \frac{1}{1 + r} [qS[j-1, l, w(j-1, L)] + (1 - q)S[j, l, w(j, L)]] \]

\[ S[j, l_{t-1}, w_t] = \alpha_j - d l - w_t + \]
\[ + \frac{1}{1 + r} [qS[j-1, l, w(j-1, L)] + (1 - q)S[j, l, w(j, L)]] \]

\[ L \text{ and } l \text{ represent current employment levels, at the aggregate and firm level respectively, while } j-1 \text{ gives business conditions opposite to } j. \text{ As in the commitment case, the shadow value } S \text{ depends on lagged employment } l_{t-1} \text{ only through the effect of the latter on the choice of current employment. When the representative firm decides on its own employment level } l \text{ it takes aggregate employment } L \text{ as given. The optimal policy is similar to the one that arises under commitment. If, in the absence of workforce adjustments, } S \text{ falls within the closed interval } [-F,0] \text{ then inaction is optimal. By contrast, if the shadow cost lies above 0 firms hire and the new employment level } l \text{ is such that the shadow value is moved downwards onto the hiring boundary } [S = 0]. \text{ Finally, if the shadow value lies below } -F \text{ firms fire and the new employment level } l \text{ is such that the shadow value is moved upwards onto the firing boundary } [S = -F]. \]

4.2 The wage strategy

In this sub-section we characterise the wage strategy and, more generally, the whole wage and employment paths in the no-commitment equilibrium.

The union chooses the wage strategy that solves the following constrained Bellman problem where, to save on notation, \( w' \) has been used in substitution of the fully specified policy variable \( w(j, L_{t-1}) \):

\[
W(j, L_{t-1}) = \max_{w'} U(w', L') + \frac{q}{1 + r} W(j-1, L') + \frac{1 - q}{1 + r} W(j, L')
\]  

\[ s.t. \quad l' = l(j, l_{t-1}, w') \quad L' = l' \quad L_{t-1} = l_{t-1} \]

\[ W(j, L_{t-1}) \text{ represents the expected discounted sum of flow payoffs } U \text{ computed along the equilibrium path. Observe that if one knew the form of the } W \text{ function the} \]
dynamic problem could be regarded as a collection of purely static problems. The union would choose \( w' \) and \( L' \) with the purpose of maximising the RHS of equation 17 under the constraint represented by firms labour demand. In graphical terms, the solution would be represented by the tangency point between labour demand and the highest indifference curve.

We notice that \( w' \) enters the maximand only through the instantaneous payoff \( U \). Thus, by the properties of \( U \), even if \( W \) is unknown we may nonetheless conclude that indifference curves become steeper as one moves upward in the wage-employment space, i.e. as one increases the wage while maintaining fixed the employment level. In turn, this implies that if the labour demand schedule moves upwards, the union chooses a point on the schedule featuring an higher wage as well as an higher employment level. We can therefore state the following result:

**Remark 1:** In a Markov equilibrium, the wage and the employment level are both normal goods for the union.

A further relevant feature of the equilibrium path is that the shadow value \( S \) must be equal to \(-F\) in all states where employment does not change from the previous period. If this were not the case, non-optimal behaviour would follow on the part of the union. More specifically, if employment does not change and the shadow value is greater than \(-F\), the union foregoes the opportunity to increase the wage by a discrete amount without paying any cost in terms of lower employment. In addition, such a wage increase is of no consequence not only in terms of current employment but also for the expected future employment and wage levels since, in a Markov equilibrium, these depend uniquely on current state variables, i.e. on current employment and business conditions. This argument also implies the following obvious corollary. In all states of the game a marginal increase of the wage from equilibrium values must reduce employment levels. Again, if this were not the case, the union would not fully exploit its monopoly position. Thus, in equilibrium, the shadow value either lies on the firing boundary or on the hiring boundary. In the first case, a wage increase leads to more firings, in the second case to less hirings. Put it differently, if the shadow value were internal between the two boundaries, a marginal increase in the wage would not lead to any reduction in employment.
Thus:

**Remark 2**

In a Markov equilibrium,

a) $S[j, l_{t-1}, w_t] = -F$ if $l[j, l_{t-1}, w_t] = l_{t-1}$

b) $S[j, l_{t-1}, w_t] = \{-F, 0\}$

Remarks 1 and 2 allow us to establish a proposition that characterises the no-commitment equilibrium.

**Proposition 3**

In a Markov equilibrium, with constant business conditions,

a) If the shadow value lies on the hiring barrier in a given period it moves to the firing barrier next period;

b) If the shadow value lies on the firing barrier in a given period it remains on the firing barrier next period.

**Proof:**

Part a) : by contradiction. Suppose the shadow value lies on the hiring barrier in period $t$ and $t+1$ while business conditions are indexed by $j$ in both periods:

$$S[j, l_{t-1}, w_t] = S[j, l_t, w_{t+1}] = 0 \quad (18)$$

Equations 16 and 18 imply that the labour demand schedules facing the union in the two periods are:

$$w_t = \frac{q}{1+r} S[j-1, l_t, w(j-1, L_t)] + \alpha_j - d l_t$$

$$w_{t+1} = \frac{1}{1+r} \{qS[j-1, l_{t+1}, w(j-1, L_{t+1})] + (1-q)S[j, l_{t+1}, w(j, L_{t+1})]\} + \alpha_j - d l_{t+1}$$

where $L_t = l_t$ and $L_{t+1} = l_{t+1}$

Recall that, by Remark 2 part b, $S = \{-F, 0\}$. If $S[j, l_{t+1}, w(j, L_{t+1})] = 0$, the second equation coincides with the first after a change in the time index from $t$ to
Thus, the two equations lead to the same schedules in the corresponding wage-employment spaces. If, by contrast, \( S[j, l_{t+1}, w(j, L_{t+1})] = -F \) the schedule in the wage-employment space at time \( t+1 \) lies below the one at time \( t \). Since employment is a normal good (Remark 1), this implies that either employment decreases from \( t \) to \( t+1 \) or it does not change. If employment decreases, the shadow value lies on the firing boundary by definition. If employment remains constant, the shadow value must lie again on the firing boundary by Remark 2 part a. Thus, in both cases, the value of \( S \) in period \( t+1 \) contradicts the statement in equation 18.

Part b): by contradiction. Suppose the shadow value lies on the firing barrier in period \( t \) and on the hiring barrier in period \( t+1 \) while business conditions are indexed by \( j \) in both periods:

\[
S[j, l_{t-1}, w_t] = -F \quad \text{and} \quad S[j, l_t, w_{t+1}] = 0
\] (19)

Equations 16 and 19 imply that the labour demand schedules facing the union in the two periods are:

\[
w_t = F + \frac{1}{1+r} q S[j_{-1}, l_t, w(j_{-1}, L_t)] + \alpha_j - d l_t
\]

\[
w_{t+1} = \frac{1}{1+r} \{ q S[j_{-1}, l_{t+1}, w(j_{-1}, L_{t+1})] + (1-q) S[j, l_{t+1}, w(j, L_{t+1})] \} + \alpha_j - d l_{t+1}
\]

where \( L_t = l_t \) and \( L_{t+1} = l_{t+1} \)

Since in equilibrium the highest value for \( S[j, l_{t+1}, w(j, L_{t+1})] \) is 0, the schedule in period \( t+1 \) lies always below the schedule in period \( t \) in the corresponding wage-employment space. In turn, since employment is a normal good (Remark 1), the union chooses an employment level \( L_{t+1} \) lower than \( L_t \), i.e. in period \( t+1 \) firms fire. This clearly contradicts the statement in equation 19.\( \diamond \)

Proposition 3 implies that firms are for most of the times on the firing boundary and that they can be on the hiring boundary only in the first period of a spell of constant business conditions. Thus, as in the commitment case, only two types of
equilibrium may arise. The first type is an equilibrium where the shadow value lies on the firing barrier at all times.\footnote{With the exception of the first period of the first spell if firms start the game with zero employment. In this case the shadow value obviously lies on the hiring barrier.} In this equilibrium, stationarity requires employment to be constant in all states whilst wages must change at business turns in order to peg the shadow value on the firing boundary. The second type is an equilibrium characterised by the shadow value on the hiring boundary in the first period of a good spell and on the firing boundary at all other times. Such an equilibrium features positive workforce adjustments, firms hire when conditions turn good from bad and fire in the opposite case. Due to its empirical relevance, in the remainder of this section we focus on this type of equilibrium.

4.3 Positive workforce adjustments

Since the shadow value is permanently on the firing barrier during a bad spell, equation 16 implies that labour demand does not move from one period to the other during the spell. This means that, along bad spells, the same wage $w_{b,nc}$ [nc: no-commitment] and the same employment level $l_{b,nc}$ are chosen in all periods. By contrast, when the state is good, the picture becomes slightly more complicated as the shadow value lies on the hiring boundary in the first period and on the firing boundary in all other subsequent periods [proposition 3]. As a consequence, the wage and the employment paths along good spells can not be characterised in simple terms as for the bad spells. We deal with this issue in proposition 4.

**Proposition 4**

In a Markov equilibrium with positive workforce adjustments, during a spell of good business conditions:

a) employment does not change from the first to the second period of the spell;

b) the wage increases by $F$ from the first to the second period of the spell;

c) employment and wages remain constant from the second period onwards.

**proof:**

Let $l_{g,nc}$ and $w_{g,nc}$ represent the employment and the wage levels in the first period of a good spell while $l'_{g,nc}$ and $w'_{g,nc}$ represent the same variables in the
second period. Notice that \( l'_{g,nc} > l_{g,nc} \) is ruled out by the fact that in the second period the shadow value is on the firing boundary (Proposition 3). Thus, contradict assertion a) and suppose \( l'_{g,nc} < l_{g,nc} \), that is suppose in the second period firms fire at a positive rate even if conditions remain good. By equation 16, the relevant labour demand schedules in the first and second period are respectively:

\[
w_{g,nc} = -\frac{F}{1+r} + \alpha_g - d \quad (20)
\]

\[
w'_{g,nc} = F - \frac{F}{1+r} + \alpha_g - d \quad (21)
\]

Both schedules embed the result that the shadow value is equal to \(-F\) in the next period no matter whether business conditions remain good or turn bad (proposition 3). Observe that the second schedule lies above the first in the wage employment space. Thus, since employment is a normal good, it follows that \( l'_{g,nc} > l_{g,nc} \). This contradicts the assumption \( l'_{g,nc} < l_{g,nc} \) and proves part a) of the proposition. More precisely, it proves that the only possible case is \( l'_{g,nc} = l_{g,nc} \).

If the employment level does not change from the first to the second period, it follows that the shift in labor demand only affects wages. Subtract equation 21 from equation 20, since the two employment levels are equal we obtain

\[
w'_{g,nc} = w_{g,nc} + F
\]

This ends the proof of point b).

Finally, notice that in the third period of a good spell the union inherits the employment level \( l_{g,nc} \) and is faced with the same schedule arising in the second period [equation 21]. Thus, the wage and employment levels of the second period are replicated in the third period and, by induction, in all other periods. This ends the proof of point c). □

Propositions 3 and 4 describe an equilibrium which exhibits many elements of the insider-outsider theory. The union increases the wage by the whole amount of firing costs after new workers have been hired and, from the second period onwards, pushes
firms onto the firing boundary. Firms, in turn, anticipate the wage increase and hire a lower number of workers. Labour demand during the hiring phase (equation 20) lies below labour demand in all subsequent periods (equation 21). Obviously, the union is harmed by firms reluctance to hire and, if possible, it would promise not to exploit the protection guaranteed by firing costs. Yet, in the absence of a commitment device, subgame perfection rules out any promise that does not result to be time-consistent. After new workers have been hired, the union can safely increase the wage by $F$ without paying any cost in terms of dismissed workers. This prevents any promise of future wage moderation and leads firms to anticipate the opportunistic behaviour of workers. In turn, the dominant ex post move for the union is to fulfill such expectations.

4.4 Computing the Markov equilibrium

After the characterisation of the Markov equilibrium with positive workforce adjustments we conclude this section with the computation of the four relevant equilibrium variables, the wage and the aggregate employment levels in the two business states: $w_{g,nc}$, $L_{g,nc}$, $w_{b,nc}$ and $L_{b,nc}$.

We start with the employment and wage levels in good times and observe that the wage in the first period $w_{g,nc}$ is determined so as to maximise the present discounted payoff flow along the spell upon taking account of the $F$ increase in wages from the second period onwards:

$$\max_{w_{g,nc}} U(w_{g,nc}, L_{g,nc}) + \frac{1-q}{r+q} U(w_{g,nc} + F, L_{g,nc})$$

subject to

$$w_{g,nc} = \frac{F}{1+r} + \alpha_g - d L_{g,nc}$$

---

\(^8\)Notice that the same aggregation rule discussed in footnote 5 applies throughout this subsection: $L_{j,nc} = l_{j,nc}$ $j = g, b$.

\(^9\)With positive workforce adjustments the level of employment in good times does not affect the payoff of the union during the following bad spell. Thus, the choice of $w_{g,nc}$ needs to maximise the payoff flow only along the good spell (equation 22).
The f.o.c. for this problem is:

\[ U_l(w_{g,nc}, L_{g,nc}) + \frac{1 - q}{r + q} U_l(w_{g,nc} + F, L_{g,nc}) = \]

\[ = d \left[ U_w(w_{g,nc}, L_{g,nc}) + \frac{1 - q}{r + q} U_w(w_{g,nc} + F, L_{g,nc}) \right] \]

(24)

Solving for \( w_{g,nc} \) and \( L_{g,nc} \) requires solving the system composed by the optimality condition 24 and the constraint 23. Notice that this system is different from the one that arises under commitment (equations 11 and 12). Thus, the ability to commit leads to different wage and employment levels during spells of good business conditions.

Guessing whether the inability to commit leads in general to lower employment levels, as argued by the insider-outsider theory, is a difficult task. Firing costs, in fact, not only move labour demand downwards (equation 23) but also bend the shape of union indifference curves in the \( w_{g,nc}-L_{g,nc} \) space (equation 22). In particular, firing costs decrease the marginal utility of \( w_{g,nc} \) and increase that of \( L_{g,nc} \) leading to an incentive to exchange lower wages for higher employment. In turn, this leads the union to counteract the negative effects of firing costs on labour demand by charging a low initial wage. What happens to employment is therefore undetermined as it results from two countervailing effects. In particular, whether firms reluctance to hiring is fully compensated by low initial wages can be assessed only upon a more detailed specification of the objective function of the union. This task is accomplished in the next section.

Turning to bad spells, since both wages and employment are constant along these spells, \( w_{b,nc} \) and \( L_{b,nc} \) may be computed by solving the simple problem below:

\[ \max_{w_{b,nc}} U(w_{b,nc}, L_{b,nc}) \]  

(25)

s.t. \( w_{b,nc} = F - \frac{1 - q}{1 + r} F + \alpha_b - d L_{b,nc} \)  

(26)

Labour demand 26 embeds the result that \( S \) is equal to \(-F\) at current time as well as in the next period if conditions persist in the bad state while it moves to the
hiring boundary if conditions change. Straightforward differentiation produces the same conditions that solve for the wage and the employment levels in the equilibrium under commitment (equations 13 and 14). Thus, wage and employment are the same during bad spells no matter whether the union is able to make a wage commitment. The ability to commit turns out to be relevant only in good times.

Finally, we identify a condition for positive adjustments to take place in equilibrium. Since in good times the union is more willing to exchange lower wages for higher employment during the hiring phase, a sufficient condition for $L_{g,nc} > L_{b,nc}$ is that labour demand 23 does not lie below labour demand 26:

$$\alpha_g - \alpha_b \geq \frac{1 + q + r}{1 + r} F$$

(27)

As for the equilibrium under commitment, this conditions requires that $F$ needs not be too high with respect to the change in the marginal revenue of labour.

We observe that, since it represents a sufficient but not a necessary condition, the inequality in equation 27 is not comparable with the one stated in equation 15 for the commitment equilibrium. Thus, we can not use the two conditions to assess whether positive adjustments are more likely in the commitment case as opposed to the no-commitment case.

In general, positive adjustments in the no-commitment equilibrium depend not only on the position of the labour demand in good and bad spells but also on the shape of the union objective function. For this reason, a more detailed description of $U$ is needed if one wants to know in which type of equilibrium positive adjustments are more likely to take place.10

5 The utilitarian case

In general, the ability to commit is relevant only if there is scope for opportunistic behaviour. In the present context, we have found that this arises during a spell

\footnote{One can easily show, for instance, that positive adjustments are more likely in the commitment case if - beyond the properties in equation 1 - the objective function also satisfies two higher order conditions: $U_{lw} \leq 0$ and $U_{ww} \geq 0$, with at least one inequality being strict. Details on this point are available from the author upon request}
of good business conditions, after new hires become insiders. However, assessing
whether the inability to commit results in a lower employment level requires some
more investigation. On the one hand, firms anticipate the wage increase and hire
less. On the other hand, the union tries to counteract low labour demand by charging
low hiring wages.

In this section we study in some detail the relative strength of these forces and,
as a consequence, the net effect of an increase in firing costs on $L_{g,nc}$. We proceed
in our analysis by assuming that the union is utilitarian, i.e. we utilize an objective
function $U$ which has been widely used in the union literature:\footnote{The obvious
reference is Oswald (1985).}

$$U(w, L) = L v(w) + (m - L) v(\overline{w})$$

In this expression, $m$ represents union membership, which we assume to be fixed,
and $(m - L)$ the number of unemployed members. The utility of each member is
given by the function $v$ whose argument is represented by the union wage $w$ for those
who happen to be employed and by the “alternative” wage $\overline{w}$ for the unemployed.

We assume that the utility function of each worker may be linear or concave
with non-negative third derivatives:\footnote{Concave utility functions with a positive third
derivative are thought to represent a reasonable
description of individual preferences. In an uncertain environment, for instance, these features
are both necessary for prudent behaviour (Blanchard and Fisher, 1989).}

$$v(w) > 0, v'(w) > 0, v''(w) \leq 0 \text{ and } v'''(w) \geq 0$$

To solve for $L_{g,c}$ substitute the utilitarian objective function in equation 11 and
combine with equation 12. Analogously, to solve for $L_{g,nc}$ substitute the function
in equation 24 and combine with equation 23. Below, we present the expressions
that result from these manipulations where, for the sake of simplicity, we have posed
$a = \frac{r+q}{1+r}$ and $R = \alpha_g - dL$:

$$\alpha_g - R = \frac{v[a \left(\frac{-1}{1+r}F + R\right) + (1 - a) \left(\frac{r}{1+r}F + R\right)] - v(\overline{w})}{v'[a \left(\frac{-1}{1+r}F + R\right) + (1 - a) \left(\frac{r}{1+r}F + R\right)]} \quad \text{(for } L_{g,c})$$
\[ \alpha_g - R = \frac{a \cdot v \left( \frac{1}{1+r} F + R \right) + (1-a) \cdot v \left( \frac{r}{1+r} F + R \right) - v(\overline{w})}{a \cdot v' \left( \frac{1}{1+r} F + R \right) + (1-a) \cdot v' \left( \frac{r}{1+r} F + R \right)} \] (for \( L_{g,nc} \))

Observe first that when \( v \) is linear \([v'' = 0]\), the two conditions coincide and the employment level is the same in the two cases no matter whether the union is able to commit or not. This result represents a notable exception to the insider-outsider proposition whereby the opportunistic behaviour of workers reduces the level of employment. Intuitively, when the utility function is linear, the union is not concerned with the actual path of wages but only with the discounted value from the whole wage flow. Thus, the union does not find it costly to charge a particularly low wage in the first period that completely counteracts the reluctance of firms towards hiring. The commitment outcome can be replicated at no cost by the union.

Suppose next that the utility function is concave with a positive third derivative. By the Jensen’s inequality, the numerator on the RHS of the expression for \( L_{g,c} \) is higher than the numerator of the expression for \( L_{g,nc} \). By contrast, the denominator is lower \((v''' > 0)\). This means that the RHS of the expression for \( L_{g,c} \) is always higher than the RHS of the expression for \( L_{g,nc} \). Further, if one regards the RHSs of the two expressions as functions of \( R \), straightforward differentiation shows that the two RHSs increase and become closer as \( R \) increases. In figure 2 we draw the RHS and the LHS of the two expressions as functions of \( R \).\(^{13}\)

Notice that the marginal revenue \( R \) is lower under commitment. Thus, we conclude that the employment level is higher under commitment, a result which is consistent with the insider-outsider mechanism. By the same argument, since firms equate the discounted flow of marginal revenues to the discounted flow of wages plus adjustment costs, wages are on average lower under commitment.

What happens when the utility function is concave? Concavity implies aversion towards anticipated sharp changes in the wage profile of the type that take place in the no-commitment case. Workers are harmed in that a constant wage profile

\(^{13}\)Curves in figure 2 are concave even if concavity is not a general feature. Depending on the shape of \( v \), convex curves are also possible. The main result of the analysis, however, holds in both circumstances, employment is higher in the commitment equilibrium also with convex curves. The CRRA utility function used to compute numbers in Table 1 below leads to concave curves.
with equal discounted value is strictly preferred to the actual one, which presents an increase of size $F$ from the second period onwards.

This fact does not explain by itself why the union chooses a lower employment level, and higher wages, in the no-commitment case. However, it is not difficult to see how this outcome results both from a lower return for the union from the employment level as well as from an higher return from the wage level. The wage shift of size $F$ from the first to the second period reduces the utility of each single employed worker and, henceforth, reduces the gain from being employed as opposed to being unemployed. This means that the union faces a lower benefit from having a large number of employed workers. This effect is captured by the numerators of the expressions above. On the other hand, since the shift is fixed in size it becomes relatively less harmful in terms of utility if wages are particularly high. It follows that the union faces an higher return from a wage increase. This effect is captured by the denominator. Thus, both channels explain why concavity leads to higher wages and lower employment levels in the no-commitment case.\footnote{Modesto and Thomas (2001) show that the no-commitment equilibrium exhibits a lower employment level even if they assume $v'' = 0$. Their result, however, is driven by a different mechanism deeply rooted in the assumption of quadratic adjustment costs.}

In Table 1 we compute the employment effect from an increase in firing costs under a standard parametrisation for the function $v$. In particular, we assume that
the utility of individuals is of the CRRA type: \( v(w) = \frac{w^{1-\gamma}}{1-\gamma} \) with \( 0 < \gamma < 1 \), this function satisfies restrictions stated in equation 28. As \( \gamma \) increases the function becomes more concave, i.e. individuals suffer more for given expected jumps in the wage path. In the table we compute the proportional change in average employment \( L_i = 0.5L_{i,g} + 0.5L_{i,b} \quad i = c, nc \) ensuing from an increase in firing cost from \( F = 1 \) to \( F = 5 \).

We observe that when the utility function is almost linear \( [\gamma = 0.1] \) the two equilibria present the same variation in average employment. In spite of the absence of a commitment, the union is capable of replicating (almost) the same employment outcome arising under commitment. In addition, the overall employment effect is close to nil as one should expect from the relatively large reversion probability \( q \) on the basis of the previous discussion on the size of the ”discounting effect”. When the curvature increases, the insider-outsider mechanism becomes more effective. We notice that average employment decreases by 3.8% in the no-commitment equilibrium if \( \gamma = 0.5 \) and by 6% if \( \gamma = 0.7 \). No employment reduction takes place in the commitment equilibrium.

15Albeit we regard table 1 just as an example we would like to remark that \( F = 5 \) is below the average wage arising in the no-commitment equilibrium (with \( \gamma = 0.7 \)). In high employment protection economies the amount of firing costs is estimated to be almost equal to the annual wage bill (Emerson 1988).

16Notice that \( F = 5 \) satisfies the necessary and sufficient condition for having positive adjustments in the commitment case (equation 15) but not the sufficient condition for the no-commitment case: (equation 27). Nevertheless, positive adjustments arise in the no-commitment case also with such a high level of firing costs. With \( \gamma = 0.7 \), for instance, \( L_{g,nc} = 2.139 \) and \( L_{b,nc} = 1.889 \).
6 Concluding remarks

We have presented a model characterised by three basic assumptions: a strategic interaction between many firms and a wage setting union, stochastic business conditions and costly labour shedding. Endogenous wages convey the mechanism described by the insider-outsider theory whilst changes in business conditions induce hiring and firing in the spirit of models of dynamic labour demand. Firing costs are shown not to affect employment in the commitment equilibrium and to reduce employment in the no-commitment equilibrium. Thus, predictions from the insider-outsider theory appear to hold also within a stochastic dynamic setting provided the focus is on the no-commitment equilibrium. By contrast, predictions from models of dynamic labour demand appear robust to the introduction of endogenous wages provided the focus is on the commitment equilibrium.

In the no-commitment equilibrium, we show that the union has an incentive to exploit the insider protection guaranteed by firing costs. This leads to a wage increase after new workers have been hired. In turn, firms anticipate such an opportunistic behaviour and hold up on the number of hirings. By contrast, when the union is able to make a commitment over future wages, the ensuing equilibrium features constant wage and employment levels along any given spell. More importantly, during good spells the wage is on average lower - and the employment level higher - when compared to no-commitment values.

We have also explored the reasons for the two different employment outcomes with and without a commitment and have come to the conclusion that a crucial role is played by the curvature of individual utility functions. This curvature in fact controls for the substitutability of two wage rates received at different points in time. When workers are only concerned with the discounted flow of wages but not with the time profile of this flow, the two outcomes coincide in terms of employment levels and overall union welfare. This happens because the union does not find it costly to charge particularly low wages during the hiring phase so as to buy the same number of jobs that arise under commitment. By contrast, when workers exhibit aversion towards sharp jumps in the wage path, buying jobs through very low initial wages is costly so that the union opts for an employment level lower than the one which arises under commitment. This conclusion elucidates a further condition for the
insider-outsider mechanism to be effective. This mechanism requires wage setters to dislike sharp variations in the wage profile.

In the introduction, we have noticed that empirical research is still inconclusive regarding the relationship between firing costs and employment. What the present paper suggests is that the institutional arrangements which characterise the wage setting process may play an important role in determining such a relationship. Corporativism and cooperative industrial relations, for instance, may help to obtain bargaining outcomes close to those which arise under commitment. The ambiguity of empirical analyses could thus be done, at least in part, to the lack of a proper account of the interaction between employment protection and the bargaining environment.
References


OECD (1999), Employment Outlook;


Appendix

Derivation of equation 10

Using the Kuhn and Tucker approach, the union problem specified in the commitment case - equations 8 and 9 - can be stated as follows:

\[
\max_{w_n,\tau, l_n,\tau, \lambda_n,\tau} L = \sum_{n=1}^{\infty} \sum_{\tau=1}^{\infty} \left\{ T[(1,1), (n, \tau)] U(w_{n,\tau}, L_{n,\tau}) + 
-\mu_{n,\tau} S_{n,\tau} - \lambda_{n,\tau} [-F - S_{n,\tau}(L_{n,\tau}; L_{n,\tau-1})] \right\}
\]

\(S_{n,\tau}\) is given by equation 7, \(\mu_{n,\tau}\) and \(\lambda_{n,\tau}\) are non-negative Kuhn-Tucker multipliers.

The f.o.c.s of this problem are:

\[
T[(1,1), (n, \tau)] U_w(w_{n,\tau}, L_{n,\tau}) = \sum_{g=1}^{\tau} T[(n,g), (n, \tau)] (\lambda_{n,g} - \mu_{n,g}) + \sum_{m=1}^{n-1} \sum_{g=1}^{\infty} T[(m,g), (n, \tau)] (\lambda_{m,g} - \mu_{m,g}) \quad \text{(for } w_{n,\tau})
\]

\[
T[(1,1), (n, \tau)] U_l(w_{n,\tau}, L_{n,\tau}) = d \sum_{g=1}^{\tau} T[(n,g), (n, \tau)] (\lambda_{n,g} - \mu_{n,g}) + d \sum_{m=1}^{n-1} \sum_{g=1}^{\infty} T[(m,g), (n, \tau)] (\lambda_{m,g} - \mu_{m,g}) \quad \text{(for } l_{n,\tau})
\]

\[
\mu_{n,\tau} \geq 0 \quad S_{n,\tau} \leq 0 \quad \mu_{n,\tau} S_{n,\tau} = 0 \quad \text{(for } \mu_{n,\tau})
\]

\[
\lambda_{n,\tau} \geq 0 \quad -F - S_{n,\tau} \leq 0 \quad \lambda_{n,\tau} [-F - S_{n,\tau}] = 0 \quad \text{(for } \lambda_{n,\tau})
\]

Equation 10 in the main text is obtained by combining the f.o.c. for \(w_{n,\tau}\) and that for \(l_{n,\tau}\).

Lemma 1
In the commitment equilibrium, if firms hire at \((n, \tau + 1)\), labour demand at \((n, \tau)\) does not lie below labour demand at \((n, \tau + 1)\) in the wage employment space.

Proof

Since firms hire at \((n, \tau + 1)\):

\[ S_{n,\tau+1}(l_{n,\tau+1}; l_{n,\tau}) = 0 \quad l_{n,\tau+1} > l_{n,\tau} \]

Substitute the latter in the labour demand equations for states \((n, \tau)\) and \((n, \tau + 1)\) that are implicitly given by equation 2:

\[
 l_{n,\tau} = 1/d \left\{ \alpha_n - w_{n,\tau} - S_{n,\tau}(l_{n,\tau}; l_{-1}) + \frac{q}{1+r}S_{n+1,1}(l_{n+1,1}; l_{n,\tau}) \right\}
\]

\[
 l_{n,\tau+1} = 1/d \left\{ \alpha_n - w_{n,\tau+1} + \frac{1-q}{1+r}S_{n,\tau+2}(l_{n,\tau+2}; l_{n,\tau+1}) + \frac{q}{1+r}S_{n+1,1}(l_{n+1,1}; l_{n,\tau+1}) \right\}
\]

Labour demand at \((n, \tau)\) can not lie above labour demand at \((n, \tau + 1)\) in the corresponding wage-employment space for the following two reasons:

a) \(-S_{n,\tau}(l_{n,\tau}; l_{-1}) \geq \frac{1-q}{1+r}S_{n,\tau+2}(l_{n,\tau+2}; l_{n,\tau+1})\) since, in equilibrium, the shadow value is non-negative.

b) \(S_{n+1,1}(l_{n+1,1}; l_{n,\tau}) \geq S_{n+1,1}(l_{n+1,1}; l_{n,\tau+1})\) since, in equilibrium, higher past employment makes firing more likely, i.e. the function \(S_{n+1,1}(\cdot; \tilde{l})\) is non-increasing with respect to \(\tilde{l}\).

Lemma 2

In the commitment equilibrium, if firms fire at \((n, \tau + 1)\), labour demand at \((n, \tau)\) does no lie above labour demand at \((n, \tau + 1)\) in the wage employment space.

Proof

similar to lemma 1

Lemma 3

In the commitment equilibrium, positive hiring and firing may only take place at the beginning of a spell of constant business conditions.
proof

We first prove that at \((n, \tau + 1)\) positive hiring can not take place. Then, we prove that the same holds for positive firing.

Take the f.o.c. for \(w_{n,\tau}\) and advance the \(\tau\)-index:

\[
T[(1, 1), (n, \tau + 1)]U_w(w_{n,\tau+1}, l_{n,\tau+1}) = \sum_{g=1}^{\tau+1} T[(n, g), (n, \tau + 1)] \left( \lambda_{n,g} - \mu_{n,g} \right) + \\
+ \sum_{m=1}^{n-1} \sum_{g=1}^{\infty} T[(m, g), (n, \tau + 1)] \left( \lambda_{m,g} - \mu_{m,g} \right)
\]

Reformulate the latter by considering that \(T[(, (n, \tau + 1)] = \frac{1}{1+q} T[, (n, \tau)]\):

\[
T[(1, 1), (n, \tau)]U_w(w_{n,\tau+1}, l_{n,\tau+1}) = \sum_{g=1}^{\tau} T[(n, g), (n, \tau)] \left( \lambda_{n,g} - \mu_{n,g} \right) + \\
\frac{1 + r}{1 - q} \left( \lambda_{n,\tau+1} - \mu_{n,\tau+1} \right) + \sum_{m=1}^{n-1} \sum_{g=1}^{\infty} T[(m, g), (n, \tau)] \left( \lambda_{m,g} - \mu_{m,g} \right)
\]

(30)

Finally, combine equation 30 with the f.o.c. for \(w_{n,\tau}\):

\[
U_w(w_{n,\tau+1}, l_{n,\tau+1}) = U_w(w_{n,\tau}, l_{n,\tau}) + \frac{1 + r}{1 - q} \frac{1}{T[(1, 1), (n, \tau)]} \left( \lambda_{n,\tau+1} - \mu_{n,\tau+1} \right)
\]

Why positive hiring can not take place at \((n, \tau + 1)\)?

Suppose, by contradiction, that positive hiring takes place at \((n, \tau + 1)\). This implies \(\mu_{n,\tau+1} > 0\) and \(\lambda_{n,\tau+1} = 0\). Thus, from the last equation:

\[
U_w(w_{n,\tau+1}, l_{n,\tau+1}) < U_w(w_{n,\tau+1}, l_{n,\tau})
\]

Positive hiring implies \(l_{n,\tau+1} > l_{n,\tau}\). Thus, since \(U_{w,l} > 0\), the former is never true if \(U_{ww} = 0\). By contrast, the former may be true if \(U_{ww} < 0\) and \(w_{n,\tau+1} > w_{n,\tau}\), i.e. if both employment and wages increase from \(\tau\) to \(\tau + 1\). This requires that labour demand at \((n, \tau + 1)\) lies above labour demand \((n, \tau)\). Lemma 1, however, rules out this occurrence.
Why positive firing can not take place at \((n, \tau + 1)\)?

Suppose, by contradiction, that positive firing takes place at \((n, \tau + 1)\). This implies \(\lambda_{n, \tau+1} > 0\) and \(\mu_{n, \tau+1} = 0\). Thus, from the last equation:

\[
U_w(w_{n, \tau+1}, l_{n, \tau+1}) > U_w(w_{n, \tau+1}, l_{n, \tau})
\]

Positive firing implies \(l_{n, \tau+1} < l_{n, \tau}\). Thus, since \(U_w, l > 0\), the former is never true if \(U_{ww} = 0\). By contrast, the former may be true if \(U_{ww} < 0\) and \(w_{n, \tau+1} < w_{n, \tau}\), i.e. if both employment and wages decrease from \(\tau\) to \(\tau + 1\). This requires that labour demand at \((n, \tau + 1)\) lies below labour demand \((n, \tau)\). Lemma, however, 2 rules out this occurrence.

**Proposition 1**

*In the commitment equilibrium,*

a) the wage and the employment level are constant within any spell;

b) the shadow value of labour is constant within any spell.

**Proof**

Proof of part a). Constant employment within spells leads immediately to constant wages through the “tangency condition” of equation ?? in the main text.

Proof of part b).

From equation 2, the equilibrium value of \(S\) is a solution to a forward looking difference equation. Constant business conditions coupled with constant wage and employment levels imply that movements of \(S\) along the spell can only result from a self-sustaining process. Thus, either \(S\) is at the steady state or it increases (decreases) as a consequence of a next period increase (decrease). However, a continuous increase/decrease is ruled out by the following argument. Since current values depend on next time values through a multiplier \(\frac{1-q}{1+r}\) lower than unit , the forward dynamics of \(S\) is explosive. This implies that \(S\) would exit the \([-F, 0]\) interval in finite time, i.e. that firms would behave in a non-optimal fashion in finite time. Clearly, this can not be part of an equilibrium.


4. Lucifora C., *Union Density and Relative Wages: Is there a Relationship?*

5. Lucifora C., Sestito P., *Determinazione del salario in Italia: una rassegna della letteratura empirica*


7. Lucifora C., Rappelli F., *Profili retributivi e carriere: un'analisi su dati longitudinali*


10. Cassuti G., Dell’Aringa C., Lucifora C., *Labour Turnover and Unionism*

11. Solimene L., *Regolamentazione ed incentivi all’innovazione nel settore delle telecomunicazioni*


15. Piccirilli G., *Monetary Business Cycles with Imperfect Competition and Endogenous Growth*

17. Lucifora C., *Rules Versus Bargaining: Pay Determination in the Italian Public Sector*

18. Piccirilli G., *Hours and Employment in a Stochastic Model of the Firm*


22. Dell’Aringa C., Vignocchi C., *Employment and Wage Determination for Municipal Workers: The Italian Case*


24. Cappellari L., *Low-pay transitions and attrition bias in Italy: a simulated maximum likelihood approach*

25. Pontarollo E., Vitali F., *La gestione del parco tecnologico elettromedicale tra outsourcing e integrazione verticale*


27. Dell’Aringa C., Lucifora C., *Inside the black box: labour market institutions, wage formation and unemployment in Italy*

28. Filippini L., Martini G., *Vertical Differentiation and Innovation Adoption*


32. Piccirilli G., *Unions and Workforce Adjustment Costs*

33. Dell’Aringa C., *The Italian Labour Market: Problems and Prospects*


35. Cappellari L., *The effects of high school choices on academic performance and early labour market outcomes*

36. Cappellari L., Jenkins S. P., *Transitions between unemployment and low pay*

37. Dell’Aringa C., Pagani L., *Collective Bargaining and Wage Dispersion*

38. Comi S., *University enrolment, family income and gender in Italy*


40. Piccirilli G., *Unions, Job Protection and Employment*

I paper sono disponibili presso:
Papers are available at:

Istituto di Economia dell'Impresa e del Lavoro
Università Cattolica del Sacro Cuore
Largo Gemelli, 1
20123 Milano (ITALY)
e-mail: ist.eil@unicatt.it
http://www.unicatt.it/istituti/EconomiaImpresaLavoro