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Alessandro Fedele   Piero Tedeschi

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Istituto di Economia dell'Impresa e del Lavoro
Facoltà di Economia
Università Cattolica del Sacro Cuore
Largo Gemelli, 1 - 20123 Milano

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Quality and Reputation: Is Competition Beneficial to Consumers?*

Alessandro Fedele[a] Piero Tedeschi[b]

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Abstract

In this paper we develop a model of product quality and firms' reputation. If quality is not verifiable and there is repeated interaction between firms and consumers, we show that reputation emerges as a means of disciplining the former to deliver high quality. In order to that, we also prove that competitive firms can extract some rent in producing high quality, thus providing a solution to Stiglitz (1989) puzzle, alternative and complementary to Hörner's (2002) one. Positive profit are generated in equilibria characterized by the emergence of a social norm which prescribes a minimum quality level. Moreover, we demonstrate that more concentrated industry structures deliver better quality and higher social and consumer welfare. This finding should induce cautiousness in enhancing competition when product quality is at stake. We derive our results in the specific context of after-sales service quality provided by insurance companies. Yet, we argue that our analysis is of general applicability.

Keywords: quality, reputation, Bertrand competition, insurance contracts.

JEL Codes: L13, D82, D81, C73.

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1 Introduction

In this paper we show that competition does not necessarily have favorable effects on consumer and social welfare when product quality is an issue. The intuition for this result is as follows. Firms with the highest market share typically have the strongest incentives to provide high quality goods: the deriving improvements in consumer welfare may offset the potentially negative effects due to a weaker competition on prices.

Our argument turns out to be correct if we are able to prove that firms extract some rent when supplying high quality goods: this has been challenged in the last 30 years. Indeed, competition should eliminate all supracompetitive gains, making it difficult for reputational forces to induce the production of high quality goods, as argued by Stiglitz (1989). Hörner (2002) provides a first answer to Stiglitz's puzzle: in a framework with heterogeneous consumers, adverse selection due to different unobservable production costs and moral hazard concerning the quality of provided goods, Hörner shows that high prices signal high quality and make competition compatible with reputation.

Our paper provides a different and complementary answer to Stiglitz's puzzle. We prove the existence of one (though not unique) equilibrium where high prices and positive profits induce firms to credibly promise high quality. This occurs since cheating (i.e. producing a lower than promised noncontractible quality level) entails the expected cost of losing future market share as consumers' response to a public random signal about firms' behavior. The higher the profits, the strongest the incentive to provide high quality. This result holds even in the presence of free entry and Bertrand competition in all equilibria characterized by the emergence of a social norm, which prevents consumers to accept very low quality services. In such a case, firms' incentive compatibility (IC, henceforth) constraint, which is binding at the equilibrium and ensures that no customer will be cheated, requires that the market share of firms is nonnegliglible and increasing with the minimum socially accepted quality level. Finally, the same IC constraint blocks further entry. Indeed, if new firms entered, they had to provide contracts which either violate the constraint, or imply a lower quality level than the socially accepted one. Since entry is blockaded, profits cannot be nought in equilibrium. Equilibria which induce a social norm allowing lower quality levels, more entry (and lower quality) is compatible with the equilibrium and this lowers profits and consumer welfare. In the limit, when the equilibrium does not entail a social norm regarding the quality level, each firm has a negligible market share, profits are nil and consumer welfare is minimum.[1]

Our setup is simplified in comparison with Hörner's, because we assume only moral hazard with homogeneous consumers and firms. Consequently, we treat reputation in a very simplified way and similar to that of Klein and Leffer (1981) and Shapiro (1983). However, with respect to Hörner, we need to introduce some specific assumptions about the public signal, as it will be explained later. As reputation is studied in a repeated Bertrand game among firms, obtaining positive equilibrium profits and, therefore, demonstrating the relevance of reputation turns out to be difficult. Yet, once accomplished the task, proving that competition could be counter-productive becomes relatively simple. Indeed, more concentrated industries can sustain higher quality (which is welfare improving), but via Bertrand competition insurance premia are bound to the minimum level satisfying the IC constraint. Hence, only positive effects of a concentrated market structure (higher quality) arise, while bad ones (higher prices) are absent.

We believe however that also other kinds of competition would deliver our result, even though somehow weakened. The reason is that the IC constraint, binding at equilibrium, implies positive profits. When many firms are active in the market, such a constraint would imply extremely low

[1] Restricted competition among agents with the threat of exclusion and replacement has been shown to represent an optimal contractual arrangement in presence of noncontractible tasks (see, e.g., Calzolari and Spagnolo, 2009, for an application to procurement policies).

quality: this would outdo the beneficial effects of lower prices arising, e.g., with Cournot competition. This finding has also important policy implications as it should induce cautiousness in advocating that competition has to be enhanced when quality is at stake.

Finally, we think that Bertrand competition is more suitable to describe competition in the sector we are going to study. Indeed, we apply our model to the analysis of a specific industry: insurance. Quality is identified with post-sale services. For instance, a company can influence the customer's welfare of, say, damage insurance by providing consultancy on various issues, legal, medical, technical, etc., and especially by paying damages promptly. Even though we think that our results are general, we focus on the insurance sector because here the quality problem is particularly severe. We clarify this point by means of an example, where we study the level of quality selected by a profit-maximizer insurance company.

Example 1 *Consider a consumer (also referred to as client or customer) with expected utility*

$$\tilde{U} \equiv pU(W - D + R, S) + (1 - p) U(W - \beta R), \tag{1}$$

where: $p \in (0,1)$ is the probability the consumer suffers a damage; U is a Bernoulli utility function with $U_W > 0 > U_{WW}$, $U_S > 0 > U_{SS}$ (subscripts W and S denote partial derivatives) and $U(x)$ is a (slightly abusive) shortcut for $U(x, 0)$; W is the consumer's initial wealth; D is the monetary value of the damage; R is the contractual level of a reimbursement provided by an insurance company (also referred to as insurer or firm); S indicates the monetary equivalent of a service, provided by the insurer, which is able to reduce material and psychological costs borne by the consumer on top of the damage D (a quick payment of the amount R is an example); βR is the premium and therefore β is the premium ratio. The role of S is to measure the quality of service provided by the insurer: the lower S, the worse the service quality.

The first order condition $\frac{\partial \tilde{U}}{\partial R} = 0$ defines the optimal insurance level R^*, i.e. the demand for insurance:

$$pU_W(W - D + R^*, S) - \beta(1 - p) U_W(W - \beta R^*) = 0. \tag{2}$$

The second order condition is satisfied since $U_{WW} < 0$. The implicit function theorem applied to (2) ensures that:

$$sign\left(\frac{\partial R^*}{\partial S}\right) = sign\left(\frac{\partial^2 \tilde{U}}{\partial W \partial S}\right) = pU_{WS}(W - D + R^*, S).$$

If $U_{WS} \equiv \frac{\partial^2 \tilde{U}}{\partial W \partial S} < 0$, then the demand for insurance decreases as the quality level of the services supplied by the company increases, i.e. R and S are substitute for the consumer.

The implication of this comparative statics result is straightforward and rather strong. Indeed, suppose the firm selects β and S to maximize the following expected profit:

$$\tilde{\Pi} \equiv [(1 - p)\beta - p] R^* - pc(S), \tag{3}$$

where $c(S)$ denotes the administrative costs in case of the provision of a service with quality S, with $c(\cdot)$ twice differentiable, $c' > 0$, $c'' > 0$, $c(0) = 0$, $c'(0) = 0$ and $c'(\infty) = +\infty$. Notice that:

$$\frac{\partial \tilde{\Pi}}{\partial S} = [(1 - p)\beta - p] \frac{\partial R^*}{\partial S} - pc'(S) < 0, \tag{4}$$

hence the firm has no incentive to provide a positive quality of the service because, on the one hand, the demand for insurance diminishes and, on the other hand, the administrative costs increase. By contrast, if $U_{WS} > 0$, then $\frac{\partial R^*}{\partial S} > 0$ and the sign of $\frac{\partial \tilde{\Pi}}{\partial S}$ is ambiguous. In this case the firm might *have an incentive to set a positive level of quality. In the remainder of the paper we focus on the case of wealth and service quality as substitute, i.e.* $U_{WS} < 0$*, which represents a more challenging starting point given the purposes of our analysis and is probably more appealing from an empirical point of view.*

The example above illustrates that monopolistic insurers have no incentive to invest in quality when wealth and service quality are substitute for the consumers, even if the latter is verifiable.

The problem of inducing high quality might be mitigated when taking into account more realistic cases. One might, for instance, expect that introducing competition would help solve the quality problem. We prove that this is true only if we assume verifiability of S. However, this seems a rather strong hypothesis in the specific case we deal with and in general when quality issues are at stake. In our case, for instance, a delay in the reimbursement can be justified in a lot of different ways which may make it difficult for a Court to understand whether it was done on purpose by a company or it was the result of the natural course of events. As a consequence, quality level cannot be contracted upon, or it can be done only to a limited extent. This amounts to say that after-sales insurance services are experience goods (Nelson, 1970) whose quality can be modified by the companies in each period, i.e. in each length of time it takes clients to learn the quality: the insurance companies may have incentives to exert minimal effort, in which case the deriving quality level is denoted with $S = 0$. We refer to it as a minimum legal standard or a value below which under-provision of quality can be easily verified by a Court.

Different institutional solutions to the deriving moral hazard problem are potentially available. For instance, some of the services could be supplied by third parties and not necessarily by the insurer. Delegation may indeed represent a good commitment device in presence of quality nonverifiability. Yet, it may also be costly for a company if moral hazard problems of the insured agent and/or of the third party, or problems of frauds, are taken into account. In these cases longer investigation by the insurance company before indemnifying the customer might be justified, with the effect of preventing the company from resorting to a delegation option. At the same time, customers often lack the competence to select and hire a suitable third party. What we usually observe, indeed, is that the reimbursement process is managed directly by the insurers, who, together with the policy, sell also legal and technical assistance in the case of damage. These premises reinforce our idea that studying the incentives for an insurance firm to supply a correct quality level of after-sales services is an important economic issue.

Related literature. As far as we know, we are the first to address the problem of quality in after-sales insurance services. Nevertheless the paper has connection, in the first place, with the literature on the administrative costs of insurance companies. Gollier (1992, 2000) indicate marketing costs, management costs and costs to audit claims as main sources of administrative expenses for insurers. Such costs are generally modelled as a function of the contractual level of reimbursement. By contrast, we model the quality level as directly influencing them. Our approach is different because we believe that quality of after-sales insurance services is closely connected with hiring experts in order to verify damages, having better call centers and/or agencies, having a policy for quick indemnity, better attorneys to assist the clients, etc.: all these aspects generate costs which increase with the service quality but, as a first approximation, do not depend on the level of reimbursement. Other papers study how administrative costs drive the optimal design of insurance contracts. Raviv (1979) and Blazenko (1985), for example, show the optimality of coinsurance above a deductible when administrative costs are a strictly convex function of the reimbursement. Their contributions help formulate the model, but they have no bearing on the interpretation of our results.

A second stream of literature which is more relevant for our paper is that on self-insurance. In a seminal paper, Ehrlich and Becker (1972) define self-insurance as investments made by clients in order to reduce the severity of damages. The problem we address becomes very similar to one of self-insurance if we interpret the lack of quality S as an additional damage borne by the consumers. This makes sense insofar as wealth and service quality are supposed to be substitute. We could then allow the clients to buy an insurance for damage D and to side-contract an insurance on the additional damage S. However, this interpretation does not add insights to our analysis, as argued in Section 2.

As mentioned before, our paper has stronger connection with the literature on the effect of reputation on product quality in industrial sectors. This literature builds on the notion of quality premia (Klein and Leffer, 1981) and it is based on the idea that in a repeated game the clients can react to a monopolist's choice of selling low quality goods by not repeating their purchase. Intuitively, this reaction constitutes a punishment for the monopolist only if providing high quality commands a profit margin, which is called quality premium. Shapiro (1983) develops a competitive version of the Klein and Leffer model by allowing free entry by firms. He argues that a seller who chooses to enter the high quality segment of the market must initially sell at less than cost to induce the consumers to buy his product. This strategy, called ex-ante investment in reputation, makes the firm's ex-ante flow of profits nought. As a consequence, if free entry is assumed the quality premium can be interpreted as a competitive return on such an investment, i.e. it is just sufficient to cover the entry and quality costs (see also Stiglitz, 1989; Cooper and Ross, 1984; Holmstrom, 1999). On this topic, the most important recent contribution is Hörner (2002), already quoted.

The remainder of the paper is organized as follows. Section 2 studies the equilibrium of a Bertrand competition among the firms when service quality is verifiable. In Section 3 we relax the verifiability assumption. In Section 4 we analyze the reputational issue. Section 5 concludes.

2 Competition with Verifiable Quality

Consider an economy with a continuum of consumers, each one characterized by the expected utility function $\tilde{U}$ in (1), and $n \geq 2$ insurance companies, each one characterized by the expected profit function $\tilde{\Pi}$ in (3).[2] In this section we deal with the benchmark situation where service quality S is verifiable, therefore also contractible. We define the first best contract as the solution to the following problem: a representative consumer maximizes his expected utility $\tilde{U}$ subject to the participation constraint of a representative insurer, whose outside option is assumed to have zero value, i.e. $\tilde{\Pi} \geq 0$. We also show that the equilibrium of a one-shot Bertrand competition game among the insurers replicates the first best.

The timing of the Bertrand game is as follows:

- insurers compete à la Bertrand, making simultaneous offers of R, β and S;
- each consumer either selects the preferred triple or buys no insurance;
- Nature selects the state of the world for each consumer (i.e. damage or no damage);
- the accepted contracts are implemented.

Lemma 1 *The Bertrand equilibrium contract when S is verifiable has the following features: (1) the insurers get zero expected profits; (2) the level of service quality is positive; (3) the consumers obtain*

[2] Throughout the paper we refer to each consumer as "he" and to each insurer as "she".

maximum expected utility. The equilibrium contract is characterized by the following conditions:

$$\begin{cases} c'\left(S^{FB}\right) = \frac{U_S\left(W-D+R^{FB},S^{FB}\right)}{U'\left(W-\beta R^{FB}\right)}, \\ U_W\left(W-D+R^{FB},S^{FB}\right) = U'\left(W-\beta R^{FB}\right), \\ (1-p)\,\beta^{FB} = p\frac{R^{FB}+c\left(S\left(\beta^{FB}R^{FB}\right)\right)}{R^{FB}}, \end{cases} \tag{5}$$

where superscript FB stands for first best.

Proof. See appendix A.1. ■

It is easy to interpret Lemma 1 with the help of the usual notions of full insurance and actuarially fair premium, when service quality is not an issue, i.e. when S does not influence the consumer's utility. Full insurance is defined as the amount of R that equates consumers' monetary wealth across states:

$$W - D + R = W - \beta R \Leftrightarrow R = \frac{D}{1+\beta}.$$

In our model, the fair rate is $\beta = \frac{p}{1-p}$ and induces the consumers to buy full insurance, as one can check by substituting it into the first order conditions. The implication of Lemma 1 can now be readily seen as standard. Indeed

$$U_W\left(W-D+R^{FB},S^{FB}\right) = U'\left(W-\beta R^{FB}\right)$$

is the usual condition of optimal insurance, i.e. the marginal utility of wealth must be equal across states. This implies that if $U_{WS}\left(W-D+R,S\right)$ were nought, the optimal contract would entail full insurance, i.e. $R = \frac{D}{1+\beta}$. By contrast, since wealth and service quality are assumed to be substitute, we observe partial insurance, i.e. R^{FB} is lower than $\frac{D}{1+\beta}$, given that the premium is unfair, i.e. $\beta^{FB} > \frac{p}{1-p}$. The result on the premium is typical of optimal insurance contracts when there are administrative costs of any type. Indeed, a fair premium would induce negative profits and hence it would not satisfy the firm's participation constraint.[3]

3 Competition with Unverifiable Quality

In this section we relax the assumption that service quality S is verifiable. As argued in the introduction, services are supplied by the insurance companies after the contracts are signed and the consumers are damaged: quality S can be observed by consumers, but it is not measurable and verifiable by third parties. This means that contracts cannot be conditioned on S. As a consequence, in a one-shot relationship the companies have an incentive to renege a promised positive quality level after signing a contract (R,β). Indeed, as $\tilde{\Pi} \equiv \left[(1-p)\,\beta - p\right]R - pc\left(S\right)$ is decreasing in S for any couple (R,β), the minimum possible level $S=0$ will be the profit-maximizing choice.

Taking into account the above reasoning, we replicate the analysis of Section 2, by studying a one-shot Bertrand competition game among the insurers, under the new hypothesis of nonverifiability of S. The time structure of the game is as in Section 2.

[3] The model presented in this section can be interpreted in another way. Since wealth and service quality are supposed to be substitute, it is natural to interpret the lack of S as an additional damage borne by the consumers. Therefore the consumers are paying a unique premium and obtain two different services. One could ask whether adding another price (the price of the service) would increase the contract efficiency. Yet, it can be proved that there is no efficiency gain in doing so. It follows that the first best contract of this section is the correct reference point for all the subsequent analysis. This alternative way to present the model highlights also its similarity to that of self insurance, where the consumers insure specific risks without resorting to an insurance company. Adding the price of the service in our model would depict a situation where the consumers buy side insurance from the same company.

Lemma 2 *The Bertrand equilibrium contract when S is not verifiable has the following features: (1) the insurers get zero expected profits; (2) the level of service quality is nought; (3) the consumers obtain the maximum expected utility subject to $S = 0$. The equilibrium contract is characterized by the following equations:*

$$\begin{cases} \hat{S} = 0, \\ \hat{R} = (1-p)\, D, \\ (1-p)\, \hat{\beta} = p. \end{cases} \tag{6}$$

Proof. See Appendix A.2. ■

By comparing contract (6) to contract (5), characterized in Lemma 1, we can verify that (i) the firms' participation constraints are satisfied with equality under both arrangements, hence their profits are nought; (ii) contract (6) entails full insurance and fair premium; (iii) the clients' expected utility is lower under the former because S is zero, whereas the utility is maximized at S^{FB}, which is generically strictly higher than zero. We can conclude that the equilibrium contract when S is not verifiable entails unexploited gains from trade.

4 Contracting and Competition with Reputation

In this section we investigate under which conditions reputational concerns may induce the firms to provide a positive level of quality under the hypothesis of nonverifiability of S, and may reduce the magnitude of unexploited gains from trade discussed at the end of Section 3. We first study the contracting problem between a representative insurer and her customers. We then analyze a Bertrand competition game among the insurers.

4.1 Contracting

We consider a repeated interaction among infinitely lived clients and insurers.[4] This requires a few extra-assumptions. First, clients have to renew the insurance also in the event the damage D does realize (one can think of a public liability insurance). Furthermore, we let 1 be the measure of clients and $\sigma_{i,t} \leq 1$ the fraction of consumers served by insurer $i = 1, ..., n$ at time $t = 0, ..., \infty$, i.e. her market share, so that $p\sigma_{i,t}$ is the fraction of damaged consumers served by insurer i at time t. In each period $t \geq 0$, contracting between insurer i and her customers takes place according to the following timing:

- a representative consumer offers a contract $\left\{R_{i,t}, \beta_{i,t}, S_{i,t}\right\}$ to his insurance company;
- insurer i either accepts the contract or refuses it;
- Nature selects the state of the world for each customer;
- insurer i selects a level of $S^A_{i,t}$ for each damaged customer, where superscript A stands for actual;
- in each period Nature selects the following public signal: with probability $\varphi\left(\tau_{i,t}, \sigma_{i,t}\right) \in [0, 1]$ *all* consumers receive a signal of bad quality, where $\tau_{i,t}$ is the share of clients who enjoy an after-damage quality level $S^A_{i,t}$ lower than the contracted $S_{i,t}$, i.e. the cheated clients;
- if customers receive a signal of bad quality, they know that insurer i cheated somebody; they then decide whether to renew the contract or not.

[4]Nothing substantial would change if the assumption of infinitely lived clients were relaxed, provided that in any period the measure of those entering the market is equal to that exiting it.

The above timing depicts a moral hazard model, where the hidden effort is the actual level of S provided by the insurers after the contract is signed. We provide some restrictions on the functional form of the public signal probability $\varphi(\tau_{i,t}, \sigma_{i,t})$.

Condition 1 *(i)* $\varphi(0, \sigma_{i,t}) = 0$*: if insurer* i *cheats no clients, no signal is conveyed; (ii)* $\varphi_\tau(\tau_{i,t}, \sigma_{i,t}) > 0$*, with* $\varphi_\tau(0,0) = 0$*, and* $\varphi_{\tau\tau}(\tau_{i,t}, \sigma_{i,t}) \geq 0$*, where subscripts* τ *(and* σ *below) denote partial derivatives: probability* φ *is increasing and nonconcave in the fraction* τ *of clients cheated; (iii)* $\varphi_{\tau\sigma}(\tau_{i,t}, \sigma_{i,t}) > 0$*: this condition has a simple interpretation which will be discussed below.*[5]

At time t the discounted value of insurer i's profit is

$$V_{i,t} \equiv \sigma_{i,t} \left[\tilde{\Pi}_{i,t} + p\tau_{i,t}\left(c(S_{i,t}) - c\left(S_{i,t}^A\right)\right)\right] + \delta\left[1 - \varphi(\tau_{i,t}, \sigma_{i,t})\right] V_{i,t+1}, \tag{7}$$

where δ is the discount factor. A trade-off is faced now by the firms: when cheating $p\tau_{i,t}\sigma_{i,t}$ consumers at any time $t \geq 0$, insurer i saves the amount $p\tau_{i,t}\sigma_{i,t}\left(c(S_{i,t}) - c\left(S_{i,t}^A\right)\right)$ on administrative costs, but she incurs the expected loss $\delta\varphi(\tau_{i,t}, \sigma_{i,t}) V_{i,t+1}$ of future profits, under the assumption that no client renews the contract when receiving the signal of bad quality. Turning to another insurer is the optimal strategy for the consumers if they believe, upon receiving the signal, that the company will cheat with probability 1. We will see that this is the equilibrium behavior.

In this context, Lemma 3 computes the parametric condition for which the insurers find it profitable not to cheat any customer, i.e. $\tau_{i,t} = 0$ for any insurer i at any time t. Such a condition defines the firms' IC constraint.

Lemma 3 *No insurer decides to cheat her customers if and only if her market share is relatively high, i.e.* $\sigma \geq \underline{\sigma}$*, where* $\underline{\sigma}$ *is the market share that satisfies with equality the following condition:*

$$\varphi_\tau(0, \sigma) \geq \frac{1-\delta}{\delta}\frac{pc(S)}{\tilde{\Pi}}. \tag{8}$$

Proof. See appendix A.3. ■

The result of Lemma 3 relies on inequality (iii) of Condition 1, $\varphi_{\tau\sigma}(\tau, \sigma) > 0$, which can be interpreted as follows. Consider an insurer who decides to cheat additional customers, i.e. to raise τ: the probability that all her customers receive the signal (in which case no consumers will renew the contract, according to the above reasoning) increases since $\varphi_\tau(\tau, \sigma) > 0$. Such a variation, in turn, rises with the market share because $\varphi_{\tau\sigma}(\tau, \sigma) > 0$. If the market share is relatively high (i.e. $\sigma \geq \underline{\sigma}$), the insurer finds it unprofitable to cheat any customer in order to save on administrative costs for such a positive effect on $V_{i,t}$ is outdone by the negative one due to the (large) probability that no consumer will renew the contract.

Before proceeding with the analysis it is useful to introduce the following

Remark 1 *The IC condition* (8) *has other implications besides the one on the market share* σ*, according to which the contract is not incentive compatible if* σ *is smaller than* $\underline{\sigma}$*, given* β*,* R *and* S*. Indeed, recalling that* $\tilde{\Pi} \equiv [(1-p)\beta - p]R - pc(S)$*, the RHS of* (8) *increases with* R *and* S *and decreases with* β *ceteris paribus, while the LHS does not depend on any of these three variables. As a consequence, the contract is not incentive compatible also when better contractual conditions are required by a consumer, given his firm's market share* σ*: either bigger* R *or* S *or smaller* β *or any combinations of the three variations. The consumer, anticipating this, is able to formulate beliefs*

[5] All the properties required for the public signal probability are satisfied by the following simple explicit formulation: $\varphi(\tau_{i,t}, \sigma_{i,t}) = \tau_{i,t} \cdot \sigma_{i,t}$.

on how the insurance company will behave in the event of a damage. Finally, the following way of rewriting the IC condition

$$\tilde{\Pi} \geq \frac{1-\delta}{\delta} \frac{pc(S)}{\varphi_\tau(0,\sigma)} \tag{9}$$

highlights its similarity to the idea of quality premium (Klein and Leffer, 1981). Indeed, condition (9) *states that the insurers must receive a positive profit on each contract in order to behave (the RHS of* (9) *is strictly higher than zero if* $S > 0$ *and* $\delta < 1$*): if profits were nought the fear of foregoing future profits would not discipline them to deliver high quality services.*

We are now able to study the second best contract, which is defined as the solution to the following problem: a representative client maximizes his single-period expected utility $\tilde{U}$ subject to a representative firm's participation constraint plus IC constraint characterized in Lemma 3.

Lemma 4 *The optimal contract with reputation when quality* S *is observable but not verifiable implies monopoly, i.e.* $\sigma = 1$*, and it is characterized by:*

$$\begin{cases} c'\left(S^{SB}\right) = \frac{\delta\varphi_\tau(0,1)}{1-\delta+\delta\varphi_\tau(0,1)} \frac{U_S\left(W-D+R^{SB},S^{SB}\right)}{U'\left(W-\beta^{SB}R^{SB}\right)}, \\ U_W\left(W-D+R^{SB},S^{SB}\right) = U'\left(W-\beta^{SB}R^{SB}\right), \\ (1-p)\,\beta^{SB} = p + p\frac{1-\delta+\varphi_\tau(0,1)}{\delta\varphi_\tau(0,1)} \frac{c\left(S^{SB}\right)}{R^{SB}}, \end{cases} \tag{10}$$

where superscript SB stands for second best.

Proof. See appendix A.4. ■

Contract (10) satisfies with equality the IC constraint. This implies, taking condition (9) with equality, that the firms' single-period profit on each contract is positive and equal to

$$\tilde{\Pi} = \frac{1-\delta}{\delta} \frac{pc(S)}{\varphi_\tau(0,\sigma)}.$$

Since firms' profits are positive and social welfare, defined as the sum of profit $\tilde{\Pi}$ of a representative insurer and utility $\tilde{U}$ of a representative consumer on a single contract, cannot be bigger with respect to the first best scenario, the consumers' utility is lower under contract (10) than under contract (5). This is due to the nonverifiability of the service quality, which imposes a positive lower bound on the level of firms' profits in order for them to behave: the consumers are simply paying the cost of moving from the first best to the second best. However, contract (10) represents a Pareto improvement with respect to that without verifiability. Indeed, contract (6) satisfies the IC constraint at $S = 0$. Yet, the consumers do not select it when reputation can be built up: this means that they are better off when choosing contract (10), but also the insurance companies are (since they make positive profits instead of zero ones). As a consequence, contract (10) dominates contract (6) and it is able to reduce the magnitude of unexploited gains from trade discussed at the end of Section 3.

In addition, the IC constraint enables us to explain why monopoly arises as the optimal market structure. Indeed, the LHS of (8) is maximum for $\sigma = 1$. On the contrary, the RHS increases with R and S and decreases with β, ceteris paribus. It follows that as σ augments the insurer finds it profitable not to cheat even for bigger values of R and S or smaller values of β, ceteris paribus. In other words, the higher an insurer's market share σ, the less binding the IC constraint (8) and, in turn, the higher the consumers' expected utility.

Before proceeding with the analysis of competition, two aspects must be noticed. First, the results of Lemma 4 can be almost replicated (as we will see in Subsection 4.2) as outcome of Bertrand competition. The only difference being that at least two insurers are needed to satisfy the whole demand: this amount to require $\sigma_i \leq \frac{1}{2}$ when firms are symmetric. All other variable s vary accordingly. Otherwise, a monopolistic company would be able to exploit its bargaining power and the contract (10) could not be implemented. Second, studying how at the optimum described by Lemma 4 the quality level S is affected by the market share σ, when the latter is taken as given, is crucial to derive the results contained in Subsection 4.2.

Lemma 5 *A necessary and sufficient condition for the quality level S to increase with the market share σ is that a number $\alpha \in [0,1]$ exists such that:*

$$\begin{array}{lll} \frac{U_S}{U_W} > \alpha \frac{U_{WS}}{U_{WW}} & \textit{and} & (a) \\ \frac{c'}{c} > -(1-\alpha)\frac{U_{WS}}{U_W} & \textit{if} \quad \frac{(1-p)}{p}\frac{U_{WW}}{U''} \leq 1 \quad \textit{or} & (b) \\ \frac{(1-p)}{p}\frac{U_{WW}}{U''}\frac{c'}{c} > -(1-\alpha)\frac{U_{WS}}{U_W} & \textit{otherwise} & (c) \end{array} \qquad (11)$$

Proof. See appendix A.5. ■

In order to provide an interpretation of the above result, we have first to clarify the meaning of various conditions in (11). To fix ideas, suppose that $\frac{(1-p)}{p}\frac{U_{WW}}{U''} \leq 1$, hence conditions (11-$a$) and (11-$b$) must hold. Condition (11-a) can be re-written as:

$$-\left.\frac{\partial W}{\partial S}\right|_{U=const} = \frac{U_S}{U_W} \geq \alpha \frac{U_{WS}}{U_{WW}} = -\alpha \left.\frac{\partial W}{\partial S}\right|_{U_W=const}.$$

The LHS is the slope of the indifference curve in plane (S, W), while the RHS is proportional to the slope (in absolute value) of the locus of point where the marginal utility of wealth is constant. Clients' allocations in the good and in the bad states are on the latter curve, since the optimal contract $\left(S^{SB}, R^{SB}, \beta^{SB}\right)$ implies constant marginal utility of wealth. It can be easily shown that if this curve were (in absolute value) flatter than the indifference curve, then the clients' utility in the bad state would be higher than that in the good one. Condition (11-a) then states that the utility in the good state cannot be much higher than that in the bad one. Condition (11-b) states that the rate of increase of quality cost, $\frac{c'}{c}$, cannot be much lower than the rate of decrease (in absolute value) of marginal utility of wealth when S increases. Condition (11-c) has a similar interpretation.

We are now able to argue the role of parameter α. To this aim we look at the two boundary cases $\alpha = 0$ and $\alpha = 1$. When $\alpha = 0$, only condition (11-b) binds by stating that quality cost is high relatively to effect of quality on the marginal utility of wealth. In this case, only firms in a concentrated market can profitably and credibly transfer utility through S instead of W. Indeed, the firms' IC constraint (8) holds only for relatively high σ. If the opposite held true, i.e. $\frac{c'}{c} \to 0$, then the constraint would hold for almost any market share level σ. As a consequence, σ would not significantly affect S. Consider now $\alpha = 1$. Condition (11-a) is the binding one and implies that the clients' utility is higher in the bad state. This means that clients value good services more than monetary transfers, given the optimal contract $\left(S^{SB}, R^{SB}, \beta^{SB}\right)$, but again only firms with high market share can profitably and credibly transfer utility through S. If the opposite held true, i.e. if quality was not important to the consumers, σ would not significantly affect S. For $\alpha \in (0,1)$, a sort of convex combination of the two conditions must hold.

From now on we assume that conditions (11-a)-(11-c) are satisfied, so that the quality of the insurance services S increases with the insurers' market share σ.

4.2 Competition

In this subsection we introduce Bertrand competition among the insurers to study the characteristics of the equilibrium contract(s). The following infinitely repeated game with free entry is played by firms and consumers:

a. before $t = 0$ the insurance companies decide whether to enter the market;

b. In each period $t \geq 0$ the insurance companies compete à la Bertrand on R_t, β_t, τ_t, σ_t and S_t: the level of quality S_t is observable but not verifiable, contrary to all of the other variables, hence it is just promised by the insurers;

c. in each period the clients either select the preferred contract or buy no insurance;

d. in each period Nature selects the state of the world for each customer;

e. in each period the insurance companies select a level of S_t for each damaged customer;

f. in each period Nature selects the public signal;

g. then the game starts again from stage b.

We solve the game by focusing on symmetric Subgame Perfect Equilibria (SPEs, henceforth) in pure strategies. Symmetry means that all the insurers choose the same market share σ. This implies that $\sigma = \frac{1}{n}$, with $n \geq 2$. Given this time structure and the object of analysis we can state the following

Claim 1 *A SPE of the game described above, where the insurers compete repeatedly à la Bertrand, S is not verifiable and reputational concerns are taken into account, displays the following features:*

(i) the equilibrium contract is as in (10)*, with the only difference that the market share is not* 1*, rather it is lower than* 50%*, i.e.* $\sigma_i \leq \frac{1}{2}$ *for any* i*;*

(ii) the insurers get positive expected profits;

(iii) the consumers' expected utility is higher than that without reputation and with unverifiable quality, but lower than that with verifiable quality;

(iv) the service quality level is positive;

(v) provided that $n \geq 2$*, any increase in the number of firms would result in a reduction of the consumers' utility.*

The precise description and the proof of the equilibrium strategies is postponed to subsequent Proposition 1. Here we wish to comment the qualitative features of the equilibrium contract. First of all, like in Hörner (2002), firms end up with positive profits, even with Bertrand competition and free entry. More importantly, the novelty of our analysis lies in showing that an increase of competition leads to a decreased consumer welfare. The reason for this result is twofold. On the one hand, the equilibrium contract is such that the IC constraint is binding. It follows that, as noticed in the discussion of Lemma 4, the softer the competition among firms in terms of number of rivals in the market, i.e. the higher σ, the higher the LHS of (8). In turn, this implies that firms offer better contractual conditions to the consumers, while maintaining a credible commitment to behave. As a consequence, the latter's expected utility increases. On the other hand, insurance premia are bound to the minimum level satisfying the IC constraint, given the assumption of Bertrand competition. With different kinds of competition our result may hold only for specific ranges of the market shares variation. Think, for example, of a Cournot game; a trade-off in this case arises: increasing the number of firms decreases quality but also prices. Which of the two effects would prevail is a matter of parameterization.

We now state formally the equilibrium strategies and the implicit beliefs which sustain the equilibrium.

Proposition 1 *There exists a SPE of the infinitely repeated game described above displaying the following features:*

1. *$\hat{n}$ denotes the equilibrium number of insurers and it is defined as*

$$\hat{n} = \max\left\{n \in N : \hat{S}(n) \geq \bar{S} > 0\right\} : \tag{12}$$

N is the set of natural numbers, $\hat{S}(n)$ is the equilibrium (promised) quality level, determined implicitly in the subsequent point 2, $\bar{S}$ is a threshold quality level whose meaning and role will be specified below, and $\hat{n}$ must be weakly higher than 2;

2. *in equilibrium all insurance companies offer a contract characterized by:*

$$\begin{cases} c'\left(\hat{S}(\hat{n})\right) = \frac{\delta\varphi_\tau\left(0,\frac{1}{\hat{n}}\right)}{1-\delta+\delta\varphi_\tau(0,\hat{n})} \frac{U_S\left(W-D+\hat{R}(\hat{n}),\hat{S}(\hat{n})\right)}{U'\left(W-\hat{\beta}(\hat{n})\hat{R}(\hat{n})\right)}, \\ U_W\left(W-D+\hat{R}(\hat{n}),\hat{S}(\hat{n})\right) = U'\left(W-\hat{\beta}(\hat{n})\hat{R}(\hat{n})\right), \\ (1-p)\hat{\beta}(\hat{n}) = p + p\frac{1-\delta+\varphi_\tau(0,\hat{n})}{\delta\varphi_\tau\left(0,\frac{1}{\hat{n}}\right)} \frac{c\left(\hat{S}(\hat{n})\right)}{\hat{R}(\hat{n})}; \end{cases} \tag{13}$$

3. *out of equilibrium, that is if $n > \hat{n}$, all insurers offer the contract of Lemma 2:*

$$\begin{cases} \hat{S} = 0, \\ \hat{R} = D(1-p), \\ (1-p)\hat{\beta} = p; \end{cases} \tag{14}$$

4. *consumers accept contract* (13) *if $n \leq \hat{n}$ and accept contract* (14) *otherwise. Moreover, they refuse any other contract.*

Proof. See appendix A.6. ■

The equilibrium contract (13) is equivalent to the second best contract with the only exception that the market share σ is $\frac{1}{\hat{n}}$ in the former and 1 in the latter. Moreover, it is characterized by a positive level of quality and an upper bound to the number of insurers. These two results are driven by implicit beliefs, an example of which is given by the following system:

$$\Pr(i \text{ cheats } |\beta_i, R_i, S_i, \sigma_i) = \begin{cases} 0 & \text{if} \quad \varphi_\tau(0,\sigma_i) \geq \frac{1-\delta}{\delta} \frac{pc(S_i)}{\tilde{\Pi}(\beta_i,R_i,S_i)} \text{ and } S_i \geq \bar{S} \\ 1 & \text{otherwise;} \end{cases} \tag{15}$$

Our argument would work even if we assumed a strictly positive probability, instead of 1, of being cheated in the second line of (15): setting it to 1 simplifies the intuition. The consumers anticipate that a company will not cheat them only if the contract proposed by any insurer i, (R_i, β_i, S_i), (i) satisfies her IC constraint for any given σ_i and (ii) provides a level of quality weakly higher than $\bar{S}$. $\bar{S}$ can be interpreted as a socially accepted quality standard, which becomes effective when the firms compete repeatedly in an infinite time horizon. On the contrary, if a firm tries to offer a quality lower than $\bar{S}$ and/or the incentive constraint is not satisfied, the consumers believe that she wants to cheat all of them, by choosing an even worse quality level setting. This lower bound on quality along with Lemma 5, which states that S decreases with the number n of insurers active in the market, drive the result on the equilibrium number of firms, $\hat{n}$.

An interesting result can be derived from Proposition 1. Recall that contract (13) satisfies with equality the incentive constraint (8) when market share σ is equal to $\frac{1}{\hat{n}}$. This implies that very competitive market structures entail low consumer and social welfare. Indeed, if the lower quality bound $\bar{S}$ decreases, the equilibrium number of firms increases according to (12) and Lemma 5. Therefore, the IC constraint implies that, ceteris paribus, firms find it profitable not to cheat only for smaller values of R and S or bigger values of β. At the same time, equilibrium profit $\tilde{\Pi}$ approximates zero as $\bar{S}$ tends to zero: the RHS of (9) is nought for $S = 0$. We argue that this negative effect of competition on social welfare would not change in moving from Bertrand to Cournot environment.

5 Concluding Remarks

In this paper we tackled the issue of the quality of after-sales services provided by insurance companies. We initially characterized the first best contract in a static context and then showed that companies have no incentive to provide a positive quality level when it is unverifiable. Finally, we considered a repeated interaction between companies and consumers in order to allow reputation emerge as a means of disciplining the former to deliver high quality. We showed that, at an equilibrium of a repeated Bertrand game among the insurers, competition turns out to be harmful for the consumers in that it increases the companies' incentive to cheat by providing zero quality after-sales services. This result hinges on the following assumption: as the market share of a company decreases, the probability the consumers receive a signal of bad quality becomes less sensitive to the number of clients cheated, hence the company finds it more profitable to reduce quality in order to save on costs. Moreover, notwithstanding the assumption of Bertrand competition, firms makes positive profits in equilibrium. This is an important ingredient to get the result of harmful competition.

In conclusion, we believe we owe a brief discussion of two separated but interconnected issues that could be problematic to our analysis: uniqueness and robustness of the equilibrium.

Uniqueness. The equilibrium depends on the implicit beliefs. There obviously are other beliefs which sustains other equilibria. Even limiting the analysis to the proposed beliefs, different equilibria are sustained, depending on the value of social standard for quality $\bar{S}$, which is not unique in our equilibrium. With lower $\bar{S}$, that is when clients expect to receive lower levels of quality, we would observe a bigger number of firms in equilibrium, lower industry profits and lower quality levels.

Robustness. Our results also depends crucially on Condition 1. The positive sign of cross-derivative $\varphi_{\tau\sigma}$ determines the IC constraint (8). If condition on the cross-derivative is reversed, i.e. if $\varphi_{\tau\sigma} < 0$, then the optimal contract with reputation when quality is observable but not verifiable would entail $\sigma = 0$, i.e. "extreme" competition. Yet, these considerations do not prevent us from formulating an important policy recommendation: when quality is an issue it is not clear that authorities should promote, without any further analysis, competition to increase social and consumer welfare.

There are several possible extensions of the current analysis. Probably the most intuitive one is considering risk-averse insurers. In our opinion, this alternative assumption might reinforce our finding on competition and consumers' welfare, at least if we also abandon the assumption of a continuum of clients. Indeed, more competition implies a smaller market share for each insurer. This implies a lower diversification of risks. Hence, small risk averse insurer are less efficient than bigger ones. Another possibility would be to study Cournot competition, instead of Bertrand. As mentioned above, this would introduce a trade-off when increasing the number of firms: lower quality, on the one hand, but lower prices, on the other hand. However, quantity competition is more reasonable in different markets from the insurance ones, being thereby an important extension to prove how applicable are our results to other industries. Furthermore, it is reasonable to expect that, when the equilibrium number of firms is relatively large, the harmful effects of competition are still present in Cournot environment, as argued above.

A Appendix

A.1 Proof of Lemma 1

The proof is divided into two parts: (i) we first compute the first best contract $\left(R^{FB}, \beta^{FB}, S^{FB}\right)$; (ii) we then show that such a contract can be obtained as the outcome of Bertrand competition.

(i) The first best contract is the solution to the following problem: a representative client maximizes the expected utility subject to the firm's participation constraint and the nonnegativity conditions. In symbols:

$$\max_{R,\beta,S} pU\left(W-D+R,S\right)+\left(1-p\right)U\left(W-\beta R\right)$$
$$s.t.\ \left[\left(1-p\right)\beta-p\right]R-pc\left(S\right)\geq 0,\ S\geq 0,\ R\geq 0,\ \beta\geq 0.$$

Ignoring the nonnegativity conditions, the first order conditions with respect to R, β and S are:

$$pU_W\left(W-D+R,S\right)-\beta\left(1-p\right)U_W\left(W-\beta R\right)+\lambda\left[\left(1-p\right)\beta-p\right]=0, \tag{16}$$
$$-R\left(1-p\right)U'\left(W-\beta R\right)+\lambda\left(1-p\right)R=0, \tag{17}$$
$$pU_S\left(W-D+R,S\right)-\lambda pc'\left(S\right)=0, \tag{18}$$

respectively, where λ is the Lagrangian multiplier of the firm's participation constraint. If $R,\beta>0$ we obtain $\lambda=U'\left(W-\beta R\right)$ from (17). As a consequence the participation constraint is binding:

$$\left[\left(1-p\right)\beta-p\right]R-pc\left(S\right)=0 \tag{19}$$

Substituting $\lambda=U'\left(W-\beta R\right)$ into (16) and (18) gives

$$U_W\left(W-D+R,S\right)=U'\left(W-\beta R\right), \tag{20}$$
$$c'\left(S\right)=\frac{U_S\left(W-D+R,S\right)}{U'\left(W-\beta R\right)}, \tag{21}$$

respectively. (21) has a solution, denoted by $S\left(\beta,R\right)$, for any level of β and R, since $c'\left(S\right)$ is increasing in S, with $c'\left(0\right)=0$, and U_S/U' is positive and decreasing in S, given $U_{SS}<0$. Applying the implicit function theorem to (21) one can prove that the partial derivative of $S\left(\beta,R\right)$ with respect to β, $S_\beta\left(\beta,R\right)$, is negative. Substituting $S\left(\beta,R\right)$ into the binding constraint and solving by β one gets

$$\beta=\frac{p}{1-p}\frac{R+c\left(S\left(\beta,R\right)\right)}{R}$$

which has a solution, $\beta\left(R\right)$, for any level of R, since the RHS is positive and decreasing in β given $S_\beta\left(\beta,R\right)<0$ and $c'\left(S\right)>0$. The implicit function theorem, applied both to $S\left(\beta,R\right)$ and $\beta\left(R\right)$, ensures that the latter is a continuous function. R ranges from 0, i.e. no insurance, to the amount $R=\frac{D}{1+\beta}$ that equates consumers' monetary wealth across states, i.e. full insurance. For $R=0$, $S=0$ is the only solution to (19), given $c\left(0\right)=0$. Therefore the LHS of (20) reduces to $U'\left(W-D\right)$, which is greater than the RHS, $U'\left(W\right)$, recalling that $U_{WS}<0$; on the contrary, for $R=\frac{D}{1+\beta}$ the LHS, $U_W\left(W-\frac{\beta}{1+\beta}D,S\right)$, is lower than the RHS, $U'\left(W-\frac{\beta}{1+\beta}D\right)$. These two results, together with the fact that $\beta\left(R\right)$ is a continuous function, ensure that at least one equilibrium does exist.

(ii) We claim that the equilibrium contract when the firms compete à la Bertrand and S is verifiable has the following characteristics: (1) it is on the same indifference surface of the clients; it replicates the first best contract, i.e. (2) expected profits of the insurance companies are nought and (3) expected utility of the clients is maximum. To prove the first claim it is sufficient to notice that if the equilibrium contracts belonged to different indifference surfaces of the clients, then at least one company would get all the consumers and might be profitably deviate by slightly modifying one or more of the three contract variables, R, β or S. To prove the second claim it is sufficient to notice that if the equilibrium contracts guaranteed positive profits to the companies, then one of them would be able to profitably deviate by slightly modifying one or more among R, β or S and getting all the consumers. Also the third claim has an analogous proof. If the utility of the consumer were not maximal under the participation constraint of the insurance company, there would be ways to undercut the competitors and increase profits: a contradiction.

A.2 Proof of Lemma 2

We first compute the optimal contract when S is not verifiable as the solution to the following problem: a representative client maximizes his utility subject to the participation constraint of a representative firm and a second constraint who incorporates the fact that service quality level is bounded to zero. The client solves then the following problem:

$$\max_{R,\beta} pU(W-D+R)+(1-p)U(W-\beta R)$$
$$s.t.\ \ [(1-p)\beta-p]R\geq 0,\ R\geq 0,\ \beta\geq 0.$$

First order conditions w.r.t to R, β and the Lagrangian multiplier λ are:

$$pU'(W-D+R)-\beta(1-p)U'(W-\beta R)+\lambda[(1-p)\beta-p]=0, \tag{22}$$

$$-R(1-p)U'(W-\beta R)+\lambda(1-p)R=0, \tag{23}$$

$$[(1-p)\beta-p]R=0, \tag{24}$$

respectively. If $R>0$, equalities (23) and (24) imply

$$\lambda=U'(W-\beta R)\ \text{ and }\ (1-p)\beta=p,$$

respectively. Substituting the previous equations into (22) gives

$$U'(W-D+R)=U'(W-\beta R),$$

Solving the above equality by R and substituting $\beta=\frac{p}{1-p}$, one gets full insurance:

$$R=D(1-p).$$

To show that contract $\left(\hat{R},\hat{\beta},\hat{S}\right)$ can be obtained as the outcome of Bertrand competition it is sufficient to repeat part (ii) of the proof of Lemma 1, by simply replacing $S=S^{FB}$ with $S=0$.

A.3 Proof of Lemma 3

First notice that $V_{i,t}$ in (7) decreases with $c\left(S_{i,t}^A\right)$, hence insurer i sets $S_{i,t}^A = 0$, i.e. she provides zero quality when cheating customers. After substituting $S_{i,t}^A = 0$, we compute the derivative of $V_{i,t}$ w.r.t. $\tau_{i,t}$:

$$\frac{\partial V_{i,t}}{\partial \tau_{i,t}} = \sigma_{i,t} pc\left(S_{i,t}\right) - \delta \varphi_\tau \left(\tau_{i,t}, \sigma_{i,t}\right) V_{i,t+1}. \tag{25}$$

If we are able to prove that $\frac{\partial V_{i,t}}{\partial \tau_{i,t}} \leq 0$ for any $\tau_{i,t} \geq 0$, then $\tau_{i,t} = 0$ is an optimum for any i and any t. First notice that

$$\frac{\partial^2 V_{i,t}}{\partial \tau_{i,t}^2} = -\delta \varphi_{\tau\tau} \left(\tau_{i,t}, \sigma_{i,t}\right) V_{i,t+1} \leq 0 \text{ for } t = 0, ..., \infty.$$

Hence a sufficient (which is also necessary in case of differentiable functions) condition for the assertion is that $\frac{\partial V_{i,t}}{\partial \tau_{i,t}} \leq 0$ at $\tau_{i,t} = 0$. We assume that the solution is stationary, i.e. $V_{i,t} = V_{i,t+1}$ for all i and all $t = 0, ...\infty$, and we then check whether such a solution is admissible. Putting $V_{i,t} = V_{i,t+1}$ in (7) and omitting subscript i one gets

$$V = \sigma \frac{\tilde{\Pi} + p\tau c\left(S\right)}{1 - \delta\left(1 - \varphi\right)}.$$

Substituting the above value into (25) and omitting subscripts i and t one gets

$$\sigma pc\left(S\right) - \delta \varphi_\tau \sigma \frac{\tilde{\Pi} + p\tau c\left(S\right)}{1 - \delta\left(1 - \varphi\right)}.$$

This expression is nonpositive in $\tau = 0$ if and only if

$$pc\left(S\right) - \delta \varphi_\tau \left(0, \sigma\right) \frac{\tilde{\Pi}}{1 - \delta} \leq 0.$$

Rearranging

$$\varphi_\tau \left(0, \sigma\right) \geq \frac{1 - \delta}{\delta} \frac{pc\left(S\right)}{\tilde{\Pi}},$$

where the LHS is increasing in σ since $\varphi_{\tau\sigma}\left(0, \sigma\right) > 0$, whilst the RHS does not depend on it.

A.4 Proof of Lemma 4

Since the model is stationary (as one can see by inspecting (7) the choice of $(\beta_t, R_t, S_t, \sigma_t)$ influences only V_t and does not have any impact on V_{t+1}). In order to characterize the second best contract we solve the problem of a representative client maximizing his single-period utility subject to a representative firm's participation and IC constraints. Notice that the former, $\tilde{\Pi} \geq 0$, is slack because it is implied by the latter (see (9)). The client's problem is hence as follows:

$$\begin{aligned} &\max_{R,\beta,S,\sigma} pU\left(W - D + R, S\right) + \left(1 - p\right) U\left(W - \beta R\right) \\ &\quad \text{s.t. } p\left(1 - \delta\right) c\left(S\right) - \delta \varphi_\tau \left(0, \sigma\right) \tilde{\Pi} \leq 0, \\ &\quad S \geq 0, \quad R \geq 0, \quad \beta \geq 0, \quad \sigma \in \left(0, 1\right]. \end{aligned} \tag{26}$$

where the first constraint is (8) after rearrangement. Considering only the IC constraint, FOCs w.r.t. R, β, S and σ are respectively:

$$pU_W(W-D+R,S)-\beta(1-p)U'(W-\beta R)+\lambda\delta\varphi_\tau(0,\sigma)[(1-p)\beta-p]=0 \tag{27}$$

$$-R(1-p)U'(W-\beta R)+\lambda\delta\varphi_\tau(0,\sigma)(1-p)R=0, \tag{28}$$

$$pU_S(W-D+R,S)-\lambda pc'(S)(1-\delta+\delta\varphi_\tau(0,\sigma))=0, \tag{29}$$

$$\lambda\delta\varphi_{\tau\sigma}(0,\sigma)\{[(1-p)\beta-p]R-pc(S)\}=0. \tag{30}$$

Equation (28) yields:

$$\lambda=\frac{U'(W-\beta R)}{\delta\varphi_\tau(0,\sigma)}, \tag{31}$$

which substituted into (27) gives

$$U_W(W-D+R,S)=U'(W-\beta R). \tag{32}$$

This is the condition of optimal insurance, that is, the marginal utility of money should be the same in all states. Substituting (31) and (32) into (29) yields

$$c'(S)=\frac{\delta\varphi_\tau(0,\sigma)}{1-\delta+\delta\varphi_\tau(0,\sigma)}\frac{U_S(W-D+R,S)}{U'(W-\beta R)}.$$

From the constraint (26) taken with equality we obtain:

$$[(1-p)\beta-p]R=pc(S)\frac{1-\delta}{\sigma\delta}pc(S)$$

which substituted in (30) yields:

$$\frac{U'(W-\beta R)}{\delta\varphi_\tau(0,\sigma)}\delta\varphi_{\tau\sigma}(0,\sigma)\frac{(1-\delta)pc(S)}{\delta\varphi_\tau(0,\sigma)}=0.$$

which is never possible since the LHS is positive for any $S>0$; hence the first order condition with respect to σ is always positive. This implies that the optimal σ is equal to 1. Finally, from constraint taken with equality we compute β:

$$\beta=\frac{p}{1-p}+\frac{pc(S)}{(1-p)R}\frac{1-\delta+\varphi_\tau(0,\sigma)}{\delta\varphi_\tau(0,\sigma)}.$$

Substituting $\sigma=1$ into the above expression we obtain the result. Relying on the same reasoning used in the proof of Lemma 1 and on the conditions specified there ensures that system (10) has at least a solution.

A.5 Proof of Lemma 5

By setting

$$\begin{aligned} W_b &= W-D+R, \\ W_g &= W-\beta R, \end{aligned}$$

the problem (26) can be transformed into the equivalent:

$$\max_{W_g,W_b,S} pU(W_b,S)+(1-p)U(W_g) \tag{33}$$

$$\text{s.t. } p(1-\delta)c(S)-\delta\varphi_\tau(0,\sigma)\tilde{\Pi}\le 0,$$

$$\tilde{\Pi}=W-W_g(1-p)-p(W_b+D)-pc(S).$$

First order conditions are:

$$\begin{cases} (1-p)\,U'(W_g) - \lambda\delta\varphi_\tau(0,\sigma)(1-p) = 0 \\ pU_W(W_b,S) - \lambda\delta\varphi_\tau(0,\sigma)\,p = 0 \\ pU_S(W_b,S) - \lambda pc'(S)(1-\delta+\delta\varphi_\tau(0,\sigma)) = 0 \\ -p(1-\delta)\,c(S) + \delta\varphi_\tau(0,\sigma)(W - W_g(1-p) - p(W_b+D) - pc(S)) = 0. \end{cases} \tag{34}$$

The bordered Hessian of the problem is:

$$H = \begin{bmatrix} (1-p)\,U'' & 0 & 0 & -\delta\varphi_\tau(1-p) \\ 0 & pU_{WW} & pU_{WS} & -\delta\varphi_\tau p \\ 0 & pU_{SW} & pU_{SS} - \lambda pc''(1-\delta+\delta\varphi_\tau) & -pc'(1-\delta+\delta\varphi_\tau) \\ -\delta\varphi_\tau(1-p) & -\delta\varphi_\tau p & -pc'(1-\delta+\delta\varphi_\tau) & 0 \end{bmatrix}$$

The second order conditions of the objective function and the constraint of problem (33) ensure that the determinant of the above matrix $\det(H)$ is negative. According to the implicit function theorem $\frac{\partial S}{\partial\sigma} = \frac{\det(H_\sigma)}{\det(H)}$, where H_σ is a matrix obtained from H by substituting the third column with the following vector

$$\begin{bmatrix} \lambda\delta\varphi_{\tau\sigma}(1-p) \\ \lambda\delta\varphi_{\tau\sigma}p \\ \lambda pc'\delta\varphi_{\tau\sigma} \\ -\delta\varphi_{\tau\sigma}(W - W_g(1-p) - p(W_b+D) - pc(S)) \end{bmatrix}$$

that, in turn, derives from differentiating the system of FOCs with respect to σ and changing the sign. It follows that $sign\frac{\partial S}{\partial\sigma} = -sign\det(H_\sigma)$ given $\det(H) < 0$, for the second order conditions.

By direct computation one gets

$$\frac{\det(H_\sigma)}{\delta p^2\varphi_{\tau\sigma}(1-p)} = \varphi_\tau\lambda\delta c'(1-\delta)\left[(1-p)\,U_{WW} + pU''\right] + U''\tilde{\Pi}\left[U_{WS}\varphi_\tau\delta - (1-\delta+\delta\varphi_\tau)\,U_{WW}c'\right].$$

Hence $\det(H_\sigma) < 0$ if and only if the RHS of the above equality is negative. From (34) we have $\frac{U_W}{\lambda} = \delta\varphi_\tau$, $\frac{U_S}{\lambda} = c'(1-\delta+\delta\varphi_\tau)$, and $\delta\varphi_\tau\tilde{\Pi} = p(1-\delta)\,c(S)$. Hence inequality $\det(H_\sigma) < 0$ can be rewritten as:

$$U_Wc'(1-\delta)\left[(1-p)\,U_{WW} + pU''\right] + U''\frac{1}{\lambda}\frac{p}{\delta\varphi_\tau}(1-\delta)\,c\,(U_{WS}U_W - U_SU_{WW}) < 0.$$

Substituting again $\frac{U_W}{\lambda} = \delta\varphi_\tau$ we obtain:

$$U_Wc'(1-\delta)\left[(1-p)\,U_{WW} + pU''\right] + U''\frac{p}{U_W}(1-\delta)\,c\,(U_{WS}U_W - U_SU_{WW}) < 0.$$

Finally, dividing everything by $U_Wc'(1-\delta)$ we obtain:

$$(1-p)\,U_{WW} + pU'' + p\frac{c}{c'}\frac{U''}{U_W}U_{WS} - p\frac{c}{c'}\frac{U''}{U_W}\frac{U_S}{U_W}U_{WW} < 0.$$

Notice that only the third term is positive while the other three are negative. Hence we can re-write our condition as follows:

$$(1-p)\,U_{WW} + pU'' - p\frac{c}{c'}\frac{U''}{U_W}\frac{U_S}{U_W}U_{WW} + (\alpha_1+\alpha_2+\alpha_3)\,p\frac{c}{c'}\frac{U''}{U_W}U_{WS} =$$

$$pU''\frac{c}{c'}\left[\left(\frac{1-p}{p}\frac{U_{WW}}{U''}\frac{c'}{c} + \alpha_3\frac{U_{WS}}{U_W}\right) + \left(\frac{c'}{c} + \alpha_2\frac{U_{WS}}{U_W}\right) - \frac{U_{WW}}{U_W}\left(\frac{U_S}{U_W} - \alpha_1\frac{U_{WS}}{U_{WW}}\right)\right] < 0,$$

with: $\alpha_1+\alpha_2+\alpha_3=1$. The above inequality is satisfied if and only if there exists a vector $(\alpha_1,\alpha_2,\alpha_3)$ such that the conditions below are satisfied:

$$\begin{cases} \frac{U_S}{U_W} > \alpha_1 \frac{U_{WS}}{U_{WW}}, \\ \frac{c'}{c} > -\alpha_2 \frac{U_{WS}}{U_W}, \\ \frac{1-p}{p}\frac{U_{WW}}{U''}\frac{c'}{c} > -\alpha_3 \frac{U_{WS}}{U_W}. \end{cases} \tag{35}$$

We can impose that α_1, α_2 and α_3 are all nonnegative. Indeed, suppose for example $\alpha_1=0$. In this case, the first condition of (35) holds because $\frac{U_S}{U_W}>0$. Considering $\alpha_1<0$ does not relax such a condition, but it makes more binding the second (or the third) condition, since $\alpha_2=1-(\alpha_1+\alpha_3)$ (or $\alpha_3=1-(\alpha_1+\alpha_2)$) decreases with α_1 for any given α_3 (or α_2). An analogous reasoning can be replicated to conclude that α_2 and α_3 should be nonnegative as well. Note also that all sides of systems (35) are nonnegative if α_1, α_2, $\alpha_3\geq 0$.

Finally, we can prove that either α_2 or α_3 can be set equal to 0 without loss of generality. Suppose that $\frac{1-p}{p}\frac{U_{WW}}{U''}>1$, in which case the LHS of the third inequality is higher than $\frac{c'}{c}$. It follows that, if the second condition holds, the third condition is satisfied for any $\alpha_3\leq\alpha_2$ and for some $\alpha_3>\alpha_2$. By setting $\alpha_2=0$, the second condition holds and the third one is satisfied for some $\alpha_3=1-\alpha_1$ larger than 0. If the opposite inequality holds, i.e. $\frac{1-p}{p}\frac{U_{WW}}{U''}<1$, a similar reasoning can be applied by setting $\alpha_3=0$. In the text we simplify notation by letting $\alpha_1=\alpha$.

A.6 Proof of Proposition 1

We solve the game backwards and start from point 4.

(i) We separate the two cases: $n\leq\hat{n}$ and $n>\hat{n}$. In the first case, contract (13) is equivalent to the second best contract with the only exception of a different market share. As a consequence, contract (13) satisfies with the equality the IC constraint (8): the clients accept it since it is the maximum they can get. To prove it, note that two possible deviations are available to any insurer i: offering a different contract with either (a) better or (b) worse conditions for the clients. In the subcase (a), each consumer, according to the equilibrium strategy, expects all the other clients to refuse this contract. In such a case, insurer i's market share is nil, $\sigma_i=0$, with the effect that the contract does not satisfy her IC constraint. Indeed, the LHS of (8) $\varphi_\tau(0,0)$ is nil by assumption, while the RHS is positive (or nil for $S=0$). The consumers anticipate that $S_i=0$ and prefer thus to turn to any other competitor. In the subcase (b) the contract satisfies strictly the IC constraint (8). Yet, no clients would accept it and they prefer any other insurance company, which offers better contractual terms.

Consider now the case of $n>\hat{n}$. If all firms offer the contract (14), the clients accept it since, given $S=0$, it is the maximum they can get. Again, two possible deviations are available to any insurer i. We focus on that ensuring a higher utility to the clients. Such a contract must promise a positive quality level, $S_i>0$, otherwise the firm's participation constraint would be violated. Yet, each client expects that no other client will accept the contract. Therefore the market share of insurer i would be nought, $\sigma=0$, and the contract would not satisfy her IC constraint. As a consequence, no client accepts the contract with $S_i>0$.

(ii) Let us now turn to points 2 and 3. We first prove that in each period $t\geq 0$ the contract (13) is an equilibrium one for any $\hat{n}\geq n\geq 2$. To this aim, first recall that the contract characterized in (13) satisfies the IC constraint (8) with equality: if all the firms offer this contract the consumers accept it, hence the former get $\tilde{\Pi}\left(\hat{\beta}(n),\hat{R}(n),\hat{S}(n)\right)>0$ on each contract stipulated at each time t. Given that we proved that any other contract would be refused by consumer there is no profitable deviation, because it would bring zero profits instead of positive ones. Now suppose that $n>\hat{n}$. In this case the consumers would accept only the contract (14) which yields zero profits. However any other contract would also bring zero profits, hence there is no strictly profitable deviation.

(iii) We finally discuss point 1. Suppose first $n < \hat{n}$ firms enter. It follows that $\hat{S}(n+1) > \bar{S}$. This means that at least an additional firm can enter and offer contract (13) with $n+1$ firms: such an offer would be accepted given (15), hence the entrant would end up with positive profits. If entrants' outside option is zero, then $n < \hat{n}$ cannot be an equilibrium of the initial entry stage. Now suppose $n > \hat{n}$ firms enter. This implies $\hat{S}(n) < \bar{S}$, hence given (15) no client believes that the entrant and the incumbents will provide $S > 0$. It follows that the firms compete à la Bertrand and offer a contract which entails the maximum expected utility for the clients with $S = 0$ and zero expected profits for the firms. Therefore, entry is not a strictly profitable strategy for an outside firm. We are able to conclude that the only equilibrium number of firms is $n = \hat{n}$.

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