Vertical Differentiation and Innovation Adoption

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Abstract

The paper investigates a duopoly model with vertical differentiation and Bertrand competition where firms choose between process or product innovation. It is shown that three equilibria in innovation adoption may arise: two symmetric equilibria, where firms select the same innovation type, and one asymmetric equilibrium, where the high (low) quality firm chooses a product (process) innovation. The determinant of these equilibria is the ratio between the costs saving effect (a lower unit cost of output due to a process innovation) and the quality effect (the savings in quality costs due to a product innovation). The asymmetric equilibrium arises because the high quality firm has greater incentives to adopt a product innovation than the low quality firm, so that it is the first to introduce it. If firms choose asymmetrically costs heterogeneity is endogenously determined: only in this case the innovation adoptions relaxe the intensity of competition between firms.

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1 Introduction

In many markets managers face a dilemma: is it better to employ the advances in knowledge and technology to produce a higher quality good or to ensure a higher rate of return by exploiting the benefits of lower unit costs? For example, the firm's product could be the provision of an Internet link with quality represented by the data transferring speed and the choice could be between increasing the speed of transferring files or reducing the unit cost of its present network. Hence the above problem can be classified as the choice between introducing a product or a process innovation. The former consists in the production of new goods, while the latter yields a cost saving benefit in the production of an existing good. This paper tackles this problem and tries to explain what factors might be important in a firm's decision whether to direct investment (e.g. R&D expenditure) towards the introduction of a product innovation or of a process innovation.

Notwithstanding the relevance of this issue there exist almost no attempts to deal with it, since the literature has usually treated the two kinds of innovation separately.[1]. Bonanno and Haworth [1998] has provided, up to now, the closest contribution to our work. They find that the *type of competitive regime* in which the firms find themselves (Cournot *vs.* Bertrand) may explain why a firm decides to adopt a product innovation and not a process innovation (and vice versa). They study this problem in a vertically differentiated duopoly and show that *if the innovator is the high quality firm* it chooses to introduce a product innovation in case of Bertrand competition and a process innovation in presence of Cournot competition.[2] On the other hand, *if the innovator is the low quality firm* and whenever the two regimes lead to different adoptions, the Bertrand competitor chooses to introduce a process innovation, while the Cournot competitor introduces a product innovation.

Three other contributions have weaker links with our work. Rosenkranz [1996]

[1] No explanations are provided about what factors might be relevant in a firm's decision to direct R&D expenditure towards product or process innovation.

[2] In general if the innovator is the high quality firm one of three things may happen: (1) both the Cournot competitor and the Bertrand competitor choose the process innovation; or (2) both select the product innovation; or (3) they make different choices.

studies, in a Cournot duopoly model with horizontal differentiation, how two competitors will optimally invest into both process and product innovation.[3] She shows that an increase in consumers' reservation price causes firms to increase R&D investments but also to shift them towards product innovation if the relative efficiency of the two types of innovation is kept constant. It is, however, not easy to understand the role of a product innovation in an horizontal differentiation framework. Battaggion and Tedeschi [1998][4] investigate a Bertrand duopoly with vertical differentiation and focus on the effects of the different types of innovation on the degree of vertical differentiation. They do not study the strategic choice of two rival firms about the innovation adoption, but show that a symmetric adoption of a product (process) innovation will decrease (increase) the degree of vertical differentiation. Lambertini and Orsini [2000] analyze the incentives to introduce a product innovation or a process innovation in a vertical differentiated monopoly (and so there is no strategic interaction).[5]

As pointed out in stating Bonanno and Haworth's results, a natural extension of the above analyzes is to consider that, in an oligopolistic environment, the choice between a product or a process innovation is taken simultaneously by all the firms in an industry. This extension may shed light on an interesting issue: firms can take symmetric or asymmetric choices about the type of innovation to introduce, an issue with potential empirical application. If the problem of which type of innovation to adopt is studied in a vertically differentiated duopoly, it may emerge that *the high quality firm* prefers a certain type of innovation, while *the low quality firm* has more incentives to introduce the alternative type of innovation. Moreover, it is important to identify the determinants of a preference towards a

[3]She develops the idea that usually firms have a portfolio of R&D projects, some more targeted at process innovations and some at product innovation, so that the optimal mix between these two types of innovation becomes a key variable in the competitive environment.

[4]Their paper is a contribution to that stream of research (e.g. Athey and Schmutzler [1995] and Eswaran and Gallini [1996]) where a process (product) innovation has the same effect on the quantity supplied by the adopting firm and on the quality of the good of a regressive product (process) innovation, i.e. of a change in technology which reduces the good's quality.

[5]They show that the social planner and the monopolist might adopt different type of innovation.

(unit) costs reduction or a quality improvement. This is the aim of the paper.

We shall think of process innovation as a reduction in the firm's production costs, so that it can be defined as *costs saving effect* on firm's efficiency. With a different approach in comparison with the usual definition adopted in the literature, we assume that a good whose quality involves infinite production costs is a non–producible good; therefore the introduction of a new good becomes possible through a reduction in the costs of its quality. Hence we define a product innovation as *the provision of a new quality good due to a reduction in the quality costs*, i.e. a *quality effect* on firm's efficiency. A model where firms strategically choose between either a process or a product innovation can also supply some additional insights about the effects of that decision on the degree of vertical differentiation, i.e. the intensity of competition, and, furthermore, can make costs heterogeneity between firms the result of their strategic choices rather than an *ex–ante* assumption.

We will show that three types of equilibria concerning the innovation game may arise, two symmetric (both firms introduce either a process innovation or a product innovation) and one asymmetric, where the high quality firm introduces a product innovation and the low quality firm a process innovation. The explanation about the determinant of the prevailing equilibria is based on the different incentives that the two firms have about adopting a product innovation: the high quality firm has higher incentives to introduce a product innovation than the low quality firm. Hence if we build an index of the impact of the two types of innovation on firms' efficiency (i.e. *the ratio between the costs saving effect and quality effect*), we will show that adopting a product innovation becomes a dominant strategy for the high quality firm for a relatively low level of the above index (i.e. the quality effect is relatively small with respect to the costs saving effect). Hence there exists a range of this index where an asymmetric equilibrium arises and firms have different costs. Some real world examples confirm that firms selling goods with different qualities follow different market strategies: high price car manufactures are usually the first to introduce new optional (e.g. CD players, satellite navigators, ABS, etc.), supermarket chains with a good reputation are the first to adopt quality standards, while hard discounts make of price reductions (through costs savings) their mission.

These equilibria in innovation adoption have also different effects on the degree of vertical differentiation. Specifically, if quality is regarded as a fixed cost (see Bonanno [1986], Motta [1993] and Shaked and Sutton [1982, 1983]), the degree of vertical differentiation decreases in both symmetric equilibria, while it increases in the asymmetric one. If quality is a fixed and also a variable cost (Champseaur and Rochet [1989], Gal–Or [1983] and Mussa and Rosen [1978]), the degree of vertical differentiation is unchanged in case of symmetric equilibria, and rises up in the asymmetric equilibrium. Hence costs heterogeneity relaxes price competition.[6]

The paper is organized as follows. Section 2 presents a three–stage duopoly model with vertical differentiation, Section 3 analyzes the strategic choice between product and process innovation, splitted in two parts: the equilibrium when quality is a fixed cost (Section 3.1), and when quality is both a fixed and variable cost (Section 3.2). Section 4 displays the effects on the degree of vertical differentiation. Section 5 presents the main results of the paper, while their proofs are reported in the Appendix.

2 The model

Consider a three–stage duopoly model where firms sell a vertically differentiated good. At $t = 1$ firms simultaneously decide whether to adopt a ProCess innovation (PC), or a ProDuct innovation (PD).[7] Hence the action set of firm i ($i = 1, 2$) at time $t = 1$ is $I_i = \{PC, PD\}$, where I stands for innovation move. This choice affects firm i costs function, since firms may face two types of costs

[6]These results are obtained in a duopoly model with Bertrand competition and vertical differentiation where the market is uncovered (as in Bonanno and Haworth, but they also apply to the covered configuration. The literature (see Choi and Shin [1992], Wauthy [1996] and Ecchia and Lambertini [1998]) has shown that the choice of the market configuration (covered or uncovered) is endogenous. A market is covered if all consumers with a positive willingness to pay for the good buy it, while it is uncovered if some consumers do not purchase the good.

[7]We rule out the possibility of choosing both types of innovation. Furthermore, the decision not to innovate is not considered since it is always dominated by introducing one of the two innovation types.

Quality as fixed cost	
no innovation	$cy + \frac{\theta^2}{2}$
process	$c\phi y + \frac{\theta^2}{2}$
product	$cy + \frac{\psi(\theta)^2}{2}$
Quality as fixed and variable cost	
no innovation	$c\theta y + \frac{\theta^2}{2}$
process	$c\phi\theta y + \frac{\theta^2}{2}$
product	$c\theta y + \frac{\psi(\theta)^2}{2}$

Table 1: Cost functions and innovation types

function: $C(y_i, \theta_i) = cy_i + \frac{\theta_i^2}{2}$ (quality is a fixed cost) and $C(y_i, \theta_i) = c\theta_i y_i + \frac{\theta_i^2}{2}$ (quality affects fixed and variable costs), where y_i is the output of firm i and θ_i its quality. Both innovations change the costs function as displayed in Table 1. We define ϕ as the *costs saving effect* (PC leads to a decrease in the marginal cost of production), and ψ as the *quality effect* (PD grants a lower marginal cost of quality). These two effects are exogenous, since they are the results of firms' decision to direct R&D activities towards either a product or a process innovation.[8] Note that before choosing which type of innovation to adopt firms have the same costs, and that costs homogeneity is maintained if they make the same type of adoption; instead in case of asymmetric adoptions they have different costs functions. The ratio $\frac{\phi}{\psi}$ can then be defined as an index of the impact of the two innovations on firms' costs functions; hence, for a given ϕ, the lower is ψ the greater is the quality effect and the higher is the above index.

At $t = 2$ the two firms select simultaneously their quality θ_i, having observed the rival's choice at $t = 1$. At $t = 3$ they choose simultaneously the price p_i, having observed the rival's quality and the innovation adoptions. Hence at the last stage we have a Bertrand subgame. We assume that firm 1 produces a good of quality θ_1 and firm 2 a good of quality θ_2, with $\theta_1 > \theta_2$, so that firm 1 (2) is labeled as the "high" ("low") quality firm.[9]

[8]Without loss of generality we assume that fixed R&D costs are equal to 0.

[9]The choice of being either the high quality firm or the low quality firm (see Herguera and Lutz [1998]) should be studied in a stage before the choice of innovation. We do not solve this stage, but we assign a label to each firm.

The market demand is specified as follows: each consumer buys only one unit of the good, and is characterized by the net utility function $U = s\theta - p$, where $s \in [0, 1]$ and p is the price paid for the good. As usual the variable s represents the consumer's willingness to pay (a taste parameter) for the good (Tirole [1988]), and is uniformly distributed over the interval $[0, 1]$. From the above and since the consumer with the lowest willingness to pay is located in 0, he/she will never buy the good, unless $p \leq 0$. Hence the market is always "uncovered" and some consumers are always out of the market. The consumer indifferent between buying the low quality good and not buying at all has a utility given by $s\theta_2 - p_2 = 0$, so that $s = \frac{p_2}{\theta_2}$. The consumer indifferent between buying the low quality good and the high quality good has a taste parameter equal to $s^* = \frac{p_1 - p_2}{\theta_1 - \theta_2}$. Hence the two firms' market demand are

$$y_1 = \left[1 - \frac{p_1 - p_2}{\theta_1 - \theta_2}\right] \tag{1}$$

$$y_2 = \left[\frac{p_1 - p_2}{\theta_1 - \theta_2} - \frac{p_2}{\theta_2}\right] \tag{2}$$

with $y_1 + y_2 < 1$.

We look for a subgame perfect equilibrium, i.e. a pair of strategies which form a Nash equilibrium in each subgame. As usual, we compute the solution by backward induction, starting from the last stage of the game, i.e. the Bertrand subgame. Given the innovation moves available to each firm and their effects on firms' costs functions, and the quality choices (θ_1, θ_2) made by the two competitors, there exist four types of Bertrand subgame, where firms have the profit functions shown in Table 2 (the case where quality is a fixed cost) and in Table 3 (the scenario where quality is also a variable cost). In the next Section we will compute the equilibrium by backward induction, starting from the Bertrand subgame.

Profit functions		
Innovation moves	π_1	π_2
$\{PC, PC\}$	$p_1y_1 - \phi c y_1 - \frac{\theta_1^2}{2}$	$p_2y_2 - \phi c y_2 - \frac{\theta_2^2}{2}$
$\{PC, PD\}$	$p_1y_1 - \phi c y_1 - \frac{\theta_1^2}{2}$	$p_2y_2 - c y_2 - \frac{\psi\theta_2^2}{2}$
$\{PD, PC\}$	$p_1y_1 - c y_1 - \frac{\psi\theta_1^2}{2}$	$p_2y_2 - \phi c y_2 - \frac{\theta_2^2}{2}$
$\{PD, PD\}$	$p_1y_1 - c y_1 - \frac{\psi\theta_1^2}{2}$	$p_2y_2 - c y_2 - \frac{\psi\theta_2^2}{2}$

Table 2: Profits in Bertrand subgame: quality as fixed cost

Profit functions		
Innovation moves	π_1	π_2
$\{PC, PC\}$	$p_1y_1 - \phi c\theta_1 y_1 - \frac{\theta_1^2}{2}$	$p_2y_2 - \phi c\theta_2 y_2 - \frac{\theta_2^2}{2}$
$\{PC, PD\}$	$p_1y_1 - \phi c\theta_1 y_1 - \frac{\theta_1^2}{2}$	$p_2y_2 - c\theta_2 y_2 - \frac{\psi\theta_2^2}{2}$
$\{PD, PC\}$	$p_1y_1 - c\theta_1 y_1 - \frac{\psi\theta_1^2}{2}$	$p_2y_2 - \phi c\theta_2 y_2 - \frac{\theta_2^2}{2}$
$\{PD, PD\}$	$p_1y_1 - c\theta_1 y_1 - \frac{\psi\theta_1^2}{2}$	$p_2y_2 - c\theta_2 y_2 - \frac{\psi\theta_2^2}{2}$

Table 3: Profits in Bertrand subgame: quality as fixed and variable cost

3 Strategic choice between process and product innovation: equilibrium analysis

We now investigate the strategic choice between product and process innovation. To identify the equilibrium concerning the adoption of a certain type of innovation, it is necessary to compute the Nash equilibrium in each possible subgame, starting from the final stage of the game. However, while an explicit solution is always achievable for the Bertrand subgames, when, by working backward, we analyze the quality subgame a general solution is not achievable. Hence we have worked out a simulation analysis which covers a sufficiently large range of parameters to generalize the results. Moreover, in all parameters configurations the same type of equilibria arise, so that the generalization seems sufficiently robust. First we compute the choice between the two types of innovation if quality is only a fixed costs.

Inn. moves	p_1^*	p_2^*
$\{PC, PC\}$	$\beta_1 + \frac{3c\phi}{4\theta_1-\theta_2}$	$\beta_2 + \frac{c\phi(2\theta_1+\theta_2)}{4\theta_1-\theta_2}$
$\{PC, PD\}$	$\beta_1 + \frac{c(1+2\phi)}{4\theta_1-\theta_2}$	$\beta_2 + \frac{c(2\theta_1+\phi\theta_2)}{4\theta_1-\theta_2}$
$\{PD, PC\}$	$\beta_1 + \frac{c(2+\phi)}{4\theta_1-\theta_2}$	$\beta_2 + \frac{c(2\phi\theta_1+\theta_2)}{4\theta_1-\theta_2}$
$\{PD, PD\}$	$\beta_1 + \frac{3c}{4\theta_1-\theta_2}$	$\beta_2 + \frac{c(2\theta_1+\theta_2)}{4\theta_1-\theta_2}$
Inn. moves	y_1^*	y_2^*
$\{PC, PC\}$	$\frac{2\theta_1-c\phi}{4\theta_1-\theta_2}$	$\frac{\theta_1(\theta_2-2c\phi)}{\theta_2(4\theta_1-\theta_2)}$
$\{PC, PD\}$	$\frac{2\theta_1(\theta_1-\theta_2)-c[(2\phi-1)\theta_1-\phi\theta_2]}{(4\theta_1-\theta_2)(\theta_1-\theta_2)}$	$\frac{\theta_1(\theta_2(\theta_1-\theta_2)+c[(1+\phi)\theta_2-2\theta_1]}{\theta_2(4\theta_1-\theta_2)(\theta_1-\theta_2)}$
$\{PD, PC\}$	$\frac{2\theta_1(\theta_1-\theta_2)-c[(2-\phi)\theta_1-\theta_2]}{(4\theta_1-\theta_2)(\theta_1-\theta_2)}$	$\frac{\theta_1(\theta_2(\theta_1-\theta_2)+c[(1+\phi)\theta_2-2\phi\theta_1]}{\theta_2(4\theta_1-\theta_2)(\theta_1-\theta_2)}$
$\{PD, PD\}$	$\frac{2\theta_1-c}{4\theta_1-\theta_2}$	$\frac{\theta_1(\theta_2-2c)}{\theta_2(4\theta_1-\theta_2)}$

Table 4: Bertrand outcomes - uncovered market, θ as fixed cost

3.1 Innovation adoptions when quality is a fixed cost

If quality only affects fixed costs, the Bertrand subgame yields the equilibrium outcomes shown in Table 4, with $\beta_1 \equiv \frac{\theta_1(2(\theta_1-\theta_2))}{4\theta_1-\theta_2}$ and $\beta_2 \equiv \frac{\theta_2(\theta_1-\theta_2)}{4\theta_1-\theta_2}$. No *a priori* ranking between the two firms' market share is possible.[10] At $t = 2$ the two firms have to solve the quality subgame. If the action profile at $t = 1$ is $\{PC, PC\}$ the two levels of quality are implicitly defined[11] by the following FOC's:

$$\frac{\partial \pi_1}{\partial \theta_1} \Rightarrow \frac{(2\theta_1 - c\phi)[c\phi(4\theta_1 - 7\theta_2) + 2(4\theta_1^2 - 3\theta_1\theta_2 + 2\theta_2^2)]}{(4\theta_1 - \theta_2)^3} = \theta_1 \quad (3)$$

$$\frac{\partial \pi_2}{\partial \theta_2} \Rightarrow \frac{\theta_1(\theta_2 - 2c\phi)[\theta_1\theta_2(4\theta_1 - 7\theta_2) + 2c\phi(4\theta_1^2 - 3\theta_1\theta_2 + 2\theta_2^2)]}{\theta_2^2(4\theta_1 - \theta_2)^3} = \theta_2 \quad (4)$$

The corresponding FOCs' for the other symmetric action profile $\{PD, PD\}$ are

$$\frac{(2\theta_1 - c)[c(4\theta_1 - 7\theta_2) + 2(4\theta_1^2 - 3\theta_1\theta_2 + 2\theta_2^2)]}{(4\theta_1 - \theta_2)^3} = \psi\theta_1 \quad (5)$$

$$\frac{\theta_1(\theta_2 - 2c\phi)[\theta_1\theta_2(4\theta_1 - 7\theta_2) + 2c\phi(4\theta_1^2 - 3\theta_1\theta_2 + 2\theta_2^2)]}{\theta_2^2(4\theta_1 - \theta_2)^3} = \psi\theta_2 \quad (6)$$

[10] Each level of output depends also upon θ_1 and θ_2, which have to be computed along the equilibrium path.

[11] For a simplified costs function with no unit costs of production Lutz [1997] obtains an analytical solution for qualities.

Similarly, we can get the corresponding FOCs' for the two asymmetric action profiles $\{PD, PC\}$ and $\{PC, PD\}$.[12]

An analytical solution to each system of FOCs' is not achievable. However, it is possible to identify a solution once a value of c has been specified.[13] The latter must guarantee non–negative profits to each firm. Numerical simulations show that $\pi_1 > 0$ *and* $\pi_2 > 0$ *iff* $0 \leq c \leq 0.0082$.[14] Within this interval two values of c have been selected: $\{0.002, 0.008\}$. The first (second) one identifies the case of low (high) costs of production. Once that a value of c has been selected it is possible to depict some qualitative aspects of the two firms' reaction functions in the quality subgame *before the innovation adoption*. First note that both firms have reaction functions with a positive slope, so that they are strategic complements in quality levels. Second, the low quality firm's reaction function is

[12] The implicit solutions if firms select $\{PD, PC\}$ are

$$\frac{\partial \pi_1}{\partial \theta_1} = \frac{\{2\theta_1(\theta_1-\theta_2)+c[\theta_1(\phi-2)+\theta_2]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2}+$$

$$+\frac{\{2(\theta_1-\theta_2)(4\theta_1^2-3\theta_1\theta_2+2\theta_2^2)-c[5\theta_2(2\theta_1-\theta_2)-8\theta_1^2+\phi(4\theta_1^2+\theta_1\theta_2-2\theta_2^2)]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2} = \psi\theta_1$$

$$\frac{\partial \pi_2}{\partial \theta_2} = \frac{\theta_1\{\theta_2(\theta_1-\theta_2)-c[2\phi\theta_1-\theta_2(1+\phi)]\}}{\theta_2^2(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2}+$$

$$+\frac{\{\theta_1\theta_2(4\theta_1-7\theta_2)(\theta_1-\theta_2)+c[\phi\theta_1(4\theta_1-3\theta_2)(2\theta_1-3\theta_2)-2\phi\theta_2^3]\}}{\theta_2^2(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2} = \theta_2$$

while if firms choose $\{PC, PD\}$

$$\frac{\partial \pi_1}{\partial \theta_1} = \frac{\{2\theta_1(\theta_1-\theta_2)-c[\theta_1(2\phi-1)-\phi\theta_2]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2}+$$

$$+\frac{\{2(\theta_1-\theta_2)(4\theta_1^2-3\theta_1\theta_2+2\theta_2^2)-c[5\phi\theta_2(2\theta_1-\theta_2)-8\phi\theta_1^2+4\theta_1^2+\theta_1\theta_2-2\theta_2^2)]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2} = \theta_1$$

$$\frac{\partial \pi_2}{\partial \theta_2} = \frac{\theta_1\{\theta_2(\theta_1-\theta_2)-c[2\theta_1-\theta_2(1+\phi)]\}}{\theta_2^2(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2}+$$

$$+\frac{\{\theta_1\theta_2(4\theta_1-7\theta_2)(\theta_1-\theta_2)+c[\theta_1(4\theta_1-3\theta_2)(2\theta_1-3\theta_2)+\phi\theta_2(4\theta_1^2+\theta_1\theta_2-2\theta_2^2)-2\theta_2^3]\}}{\theta_2^2(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2} = \psi\theta_2$$

[13] Numerical computations have been performed using the program "Maple".

[14] As in Bonanno and Haworth, unit costs of production are subject to restrictions in order to have both firms with non–negative profits. Note that if the two products are sold at unit costs, i.e. $p_1 = p_2 = c$, firms make negative profits. Hence, since quality is a fixed cost, unit costs must be sufficiently small in order to ensure positive profits to both firms.

steeper than the high quality firm's reaction function: hence the low quality firm tends to react more to a change in quality of its rival than the high quality firm.

The first step in finding numerical solutions consists in fixing a value for ϕ, i.e. the costs saving effect. Three cases are investigated: $\phi \in \{0.1, 0.5, 0.9\}$. They highlight the solutions of the innovation adoption game in presence of different levels of the costs saving effect. More precisely, $\phi = 0.1$ corresponds to the case where the costs saving effect is particularly strong. For a given combination of $\{c, \phi\}$, we first derive some qualitative effects on the two firms' reaction functions in the quality subgame. If both firms introduce a process innovation their reaction functions become steeper than the pre–innovation ones: this implies than if both firms adopt a costs saving innovation they will react more to a rival's change in its quality than in the pre–innovation case. The same effects arise if the high quality firm choose the process innovation and the low quality firm the product innovation. If instead the high quality firm introduces a product innovation and its rival a process innovation, the high quality firm's reaction function becomes flatter (i.e. it reacts less to a change in its rival's quality), as well as the low quality firm's reaction function. Last, if both firms introduce a product innovation the high quality firm's reaction function becomes flatter, while the low quality firm's reaction function becomes steeper: hence the high quality firm reacts more to a change in its rival's quality, while the low quality firm reacts less.

We have now to identify the Nash equilibria in the innovation game. First we compute each firm's best reply, starting from firm 1, the high quality firm. Table 15 in the Appendix reports the numerical solutions if the low quality firm chooses PC. Note that if the high quality firm selects PC then $\theta_1^* = 0.2523$, $\theta_2^* = 0.0471$, $\pi_1^* = 0.023$ and $\pi_2^* = 0.0002$. Figure $1(a)$ shows firm 1's profits as function of ψ. The greater the quality effect (i.e. the smaller is ψ), the higher is the high firm profit if it adopts PD. On the other hand, for a given value of ϕ (i.e. the costs saving effect), firm 1's profit is fixed if it chooses PC even if the quality effect rises up (the horizontal line). Hence, as displayed, there exists a value of ψ, labeled as ψ_1 and equal to 0.9855, such that only if $\psi_1 \leq \psi \leq 1$ PC

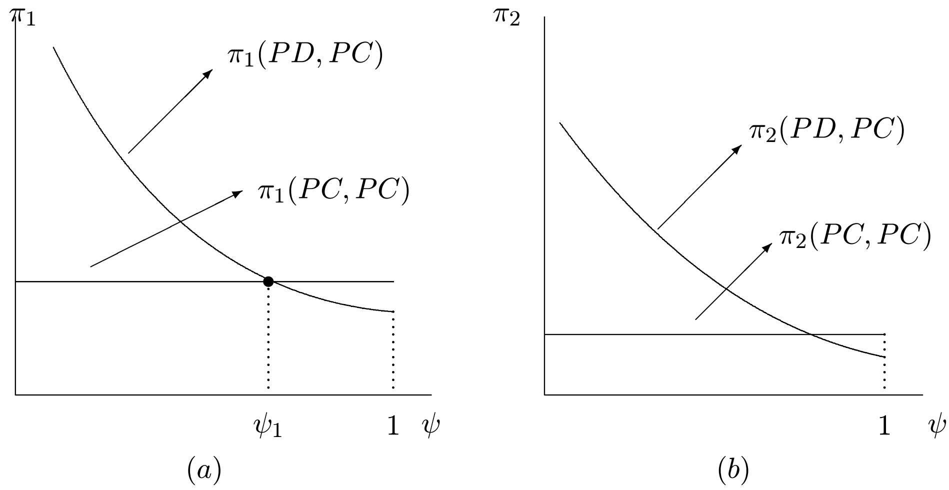

Figure 1: (a) High quality firm's profits if the low quality firm chooses PC. (b) Effects on the low quality firm's profits of the high quality firm's adoptions

dominates PD. Hence firm 1's best reply to PC is the following:

$$\begin{cases} (PC, PC) & \text{if} \quad \psi_1 \leq \psi \leq 1 \\ (PD, PC) & \text{if} \quad 0 \leq \psi < \psi_1 \end{cases} \tag{7}$$

Note that, as shown in Figure 1(b), even if the low quality selects PC, its profit increases as ψ shrinks if its rival has chosen PD. The intuition of the latter is the following: the enhancements in firm 1's quality due to the quality effect allow firm 2 to increase its quality as well, so that also its profit increases. Figure 2(a) shows firm 1's profits as a function of ψ if its rival adopts PD (the numerical solutions are reported in Table 16 in the Appendix). It is interesting to point out that its profit decreases if the quality effect rises up (i.e. profit increases with ψ) if it adopts PC and its competitor PD; the adoption profile (PC, PD) gets the two firms close together in profits terms. The intuition is that the low quality firm increases its quality by choosing PD, pushing up its market share and reducing y_1. Hence in case of asymmetric adoption firm 2 always gets a profit increase, while the high quality firm obtains a positive shift in its profitability

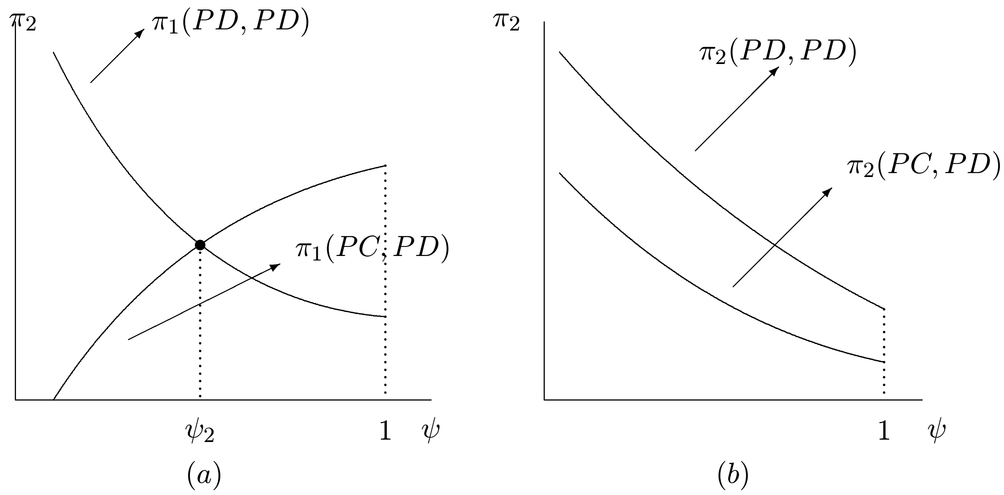

Figure 2: (a) High quality firm's profits if the low quality firm chooses PD. (b) Effects on the low quality firm's profits of the high quality firm's adoptions

only if it selects PD. As before, there exists a critical value of the quality effect ψ such that firm 1's best reply to PD swaps from one type of innovation to another. If we label this critical value as ψ_2 (equal to 0.9849), we can write the following best reply:

$$\begin{cases} (PC, PD) & \text{if} \quad \psi_2 < \psi \leq 1 \\ (PD, PD) & \text{if} \quad 0 \leq \psi \leq \psi_2 \end{cases} \tag{8}$$

Moreover, Figure 2(b) displays that firm 2's profits are always increasing with the quality effect once it selects PD, independently on its rival innovation choice, and that they are higher if the high quality firm also adopts a product innovation.

We have now to identify firm 2's best replies. If firm 1 selects PC, firm 2's profits as a function of ψ, for any $\phi \in [0, 1]$, are those shown in Figure 3(a) (the results are shown in Table 17).[15] There exists a unique value of ψ (i.e. ψ_3, equal

[15] If the two firms play (PC, PC) quality and profit outcomes are $\theta_1^* = 0.2523$, $\theta_2^* = 0.0471$, $\pi_1^* = 0.023$ and $\pi_2^* = 0.0002$.

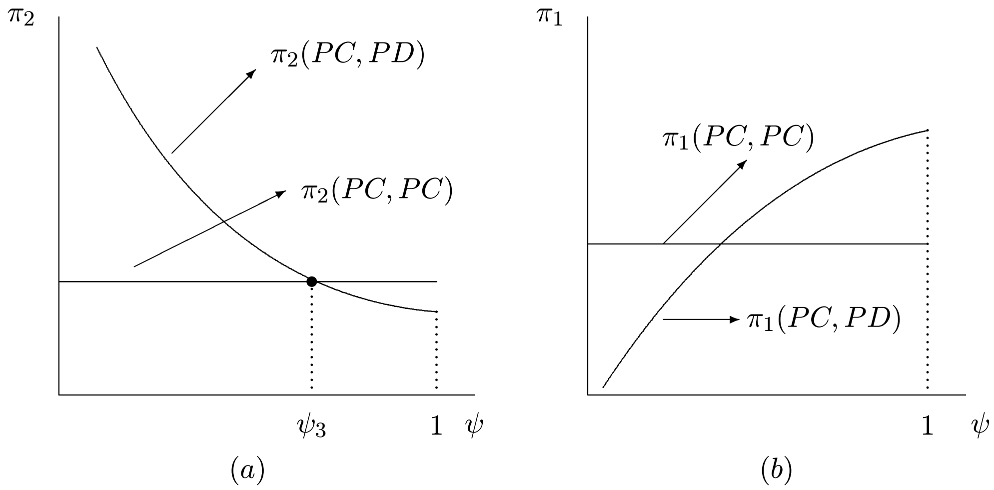

Figure 3: a) Low quality firm's profits if the high quality firm chooses PC. (b) Effects on the high quality firm's profits of the low quality firm's adoptions

to 0.886) such that the two innovation types yield the same profit level. Hence its best reply if the high quality firm adopts PC is

$$\begin{cases} (PC, PC) & \text{if} \quad \psi_3 \leq \psi \leq 1 \\ (PC, PD) & \text{if} \quad 0 \leq \psi < \psi_3 \end{cases} \tag{9}$$

Again, if the high quality firm selects PC and its competitor PD its profit shrinks as the quality effect increases, as shown in Figure 3(b). Last, Figure 4 displays firm 2's profits as a function of ψ when its rival adopts PD. As in all the other cases discussed above, there is only one value of ψ, labeled as ψ_4 and equal to 0.8913, where profits equality holds for the two types of innovation. Hence firm 2's best reply to PD is

$$\begin{cases} (PD, PC) & \text{if} \quad \psi_4 < \psi \leq 1 \\ (PD, PD) & \text{if} \quad 0 \leq \psi \leq \psi_4 \end{cases} \tag{10}$$

Conditions (7)–(10) identify the Nash equilibrium in the innovation game. Hence, if we consider the index of the impact of the two types of innovation on firms' costs functions, that is $\frac{\phi}{\psi}$, it is straightforward to claim the following, for $\phi = 0.9$:

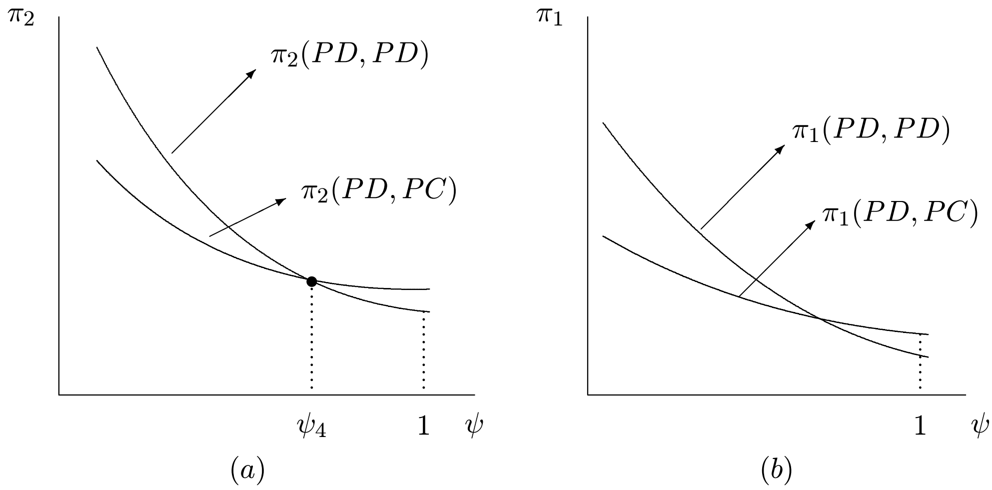

Figure 4: a) Low quality firm's profits if the high quality firm chooses PD. (b) Effects on the high quality firm's profits of the low quality firm's adoptions

Claim 1 *When quality is a fixed cost and $\phi = 0.9$ the innovation game has three types of equilibria:*
i. $\{PC, PC\}$ *if* $\frac{\overline{\phi}}{\psi} \in \quad [0.9, 0.9132]$;
ii. $\{PD, PC\}$ *if* $\frac{\overline{\phi}}{\psi} \in \quad]0.9132, 1.01[$;
iii. $\{PD, PD\}$ *if* $\frac{\overline{\phi}}{\psi} \in \quad [1.01, \infty]$;

Claim 1 shows that the determinant of the equilibrium in the innovation game is the ratio between the costs saving effect (ϕ) and the quality effect (ψ). If this ratio is smaller than 1 and the quality effect is small (i.e. $\psi \to 1$) both firms will prefer to adopt a process innovation. As $\frac{\psi}{\phi}$ rises (keeping ϕ fixed) the incentives to adopt a product innovation increase. However the high quality firm is the first to adopt it, and there exists an interval of the $\frac{\phi}{\psi}$ index where the low quality firm still finds profitable to benefit from a unit costs reduction and not from a product innovation. Last, if the quality effect becomes sufficiently large both firms introduce a product innovation. Note that when the equilibrium is (PD, PC) the two competitors have costs heterogeneity. Table 5 reports the

					Nash Equilibrium, ψ–interval		
(c, ϕ)	ψ_1	ψ_2	ψ_3	ψ_4	PC, PC	PD, PC	PD, PD
(0.008,0.9)	0.9855	0.9849	0.886	0.8913	0.9855–1	0.8913–0.9855	0–0.8913
(0.008,0.5)	0.9301	0.9279	0.552	0.616	0.9301–1	0.616–0.9301	0–0.616
(0.008,0.1)	0.8837	0.8761	0.3303	0.4631	0.8837–1	0.4631–0.8837	0–0.4631
(0.002,0.9)	0.99625	0.99624	0.96261	0.9632	0.99625–1	0.9632–0.99625	0–0.9632
(0.002,0.5)	0.9816	0.9814	0.8278	0.8408	0.9816–1	0.8408–0.9816	0–0.8408
(0.002,0.1)	0.9674	0.967	0.7126	0.7464	0.9674–1	0.7464–0.9674	0–0.7464

Table 5: Nash equilibria in innovation adoption when quality is a fixed cost

ψ–intervals, for each (c, ϕ) combination considered, where the above equilibria arise, and it underscores that the greater the costs saving effect the larger is the ψ–interval where the asymmetric equilibrium in innovation adoption prevails.

3.2 Innovation adoptions when quality is a fixed and variable cost

In this Section we apply the same procedure adopted in Section 3.1 to the case where quality is a fixed and variable cost. The Bertrand outcomes are shown in Table 6, with $\zeta_1 \equiv 2(\theta_1 - \theta_2)$ and $\zeta_2 \equiv (\theta_1 - \theta_2)$. The solutions, by backward induction, in the quality subgame in case of symmetric adoptions are those presented in Table 7, while only an implicit solution can be identified if firms select

Inn. moves	p_1^*	p_2^*
$\{PC, PC\}$	$\frac{\theta_1[\zeta_1+c\phi(2\theta_1+\theta_2)]}{4\theta_1-\theta_2}$	$\frac{\theta_2[\zeta_2+3c\phi\theta_1]}{4\theta_1-\theta_2}$
$\{PC, PD\}$	$\frac{\theta_1[\zeta_1+c(2\phi\theta_1+\theta_2)]}{4\theta_1-\theta_2}$	$\frac{\theta_2[\zeta_2+c\theta_1(2+\phi)]}{4\theta_1-\theta_2}$
$\{PD, PC\}$	$\frac{\theta_1[\zeta_1+c(2\theta_1+\phi\theta_2)]}{4\theta_1-\theta_2}$	$\frac{\theta_2[\zeta_2+c\theta_1(2\phi+1)]}{4\theta_1-\theta_2}$
$\{PD, PD\}$	$\frac{\theta_1[\zeta_1+c(2\theta_1+\theta_2)]}{4\theta_1-\theta_2}$	$\frac{\theta_2[\zeta_2+3c\theta_1]}{4\theta_1-\theta_2}$
Inn. moves	y_1^*	y_2^*
$\{PC, PC\}$	$\frac{2\theta_1(1-c\phi)}{(4\theta_1-\theta_2)}$	$\frac{\theta_1(1-c\phi)}{(4\theta_1-\theta_2)}$
$\{PC, PD\}$	$\frac{\theta_1[\zeta_1-c[2\phi\theta_1-\theta_2(1+\phi)]]}{(4\theta_1-\theta_2)(\theta_1-\theta_2)}$	$\frac{\theta_1[\zeta_1+c[(\phi-2)\theta_1+\theta_2]]}{\theta_2(4\theta_1-\theta_2)(\theta_1-\theta_2)}$
$\{PD, PC\}$	$\frac{\theta_1[\zeta_1-c[2\theta_1-\theta_2(1+\phi)]]}{(4\theta_1-\theta_2)(\theta_1-\theta_2)}$	$\frac{\theta_1[\zeta_1+c[\theta_1(1-2\phi)+\phi\theta_2]]}{(4\theta_1-\theta_2)(\theta_1-\theta_2)}$
$\{PD, PD\}$	$\frac{2\theta_1(1-c)}{(4\theta_1-\theta_2)}$	$\frac{\theta_1(1-c)}{(4\theta_1-\theta_2)}$

Table 6: Bertrand outcomes - uncovered market, θ as fixed and variable cost

different types of innovation.[16]

The simulation analysis for the innovation game under this costs configuration, performed for the same parameters of the previous Section, yields the results shown in Table 8. For instance, if $c = 0.008$ and $\phi = 0.9$ then $\psi_1 \cong \psi_2 \cong 0.99625$, $\psi_3 = 0.99171, \psi_4 \cong 0.99173$, and it is possible to claim the following.

[16] If the two firms choose $\{PD, PC\}$ the implicit solutions are

$$\frac{\partial \pi_1}{\partial \theta_1} = \frac{\theta_1\{2(\theta_1-\theta_2)-c[2\phi\theta_1-\theta_2(1+\phi)]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2}+$$

$$+\frac{\{2(\theta_1-\theta_2)(4\theta_1^2-3\theta_1\theta_2+2\theta_2^2)-c[\theta_2(4\theta_1^2+\theta_1\theta_2-2\theta_2^2)+\phi(\theta_1(4\theta_1-3\theta_2)(2\theta_1-3\theta_2)-2\theta_2^3)\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2} = \theta_1$$

$$\frac{\partial \pi_2}{\partial \theta_2} = \frac{\theta_1^2\{(\theta_1-\theta_2)+c[(\phi-2)\theta_1+\theta_2]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2}+$$

$$+\frac{\{(\theta_1-\theta_2)(4\theta_1-7\theta_2)+c[\phi(4\theta_1^2+\theta_1\theta_2-2\theta_2^2)+5\theta_2(2\theta_1-\theta_2)+8\theta_1^2)\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2} = \psi\theta_2$$

and if they select $\{PC, PD,\}$ we have

$$\frac{\partial \pi_1}{\partial \theta_1} = \frac{\theta_1\{2(\theta_1-\theta_2)-c[2\theta_1-\theta_2(1+\phi)]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2}+$$

$$\frac{\{2(\theta_1-\theta_2)(4\theta_1^2-3\theta_1\theta_2+2\theta_2^2)-c[\phi\theta_2(4\theta_1^2+\theta_1\theta_2-2\theta_2^2)+\theta_1(4\theta_1-3\theta_2)(2\theta_1-3\theta_2)-2\theta_2^3)]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2} = \psi\theta_1$$

$$\frac{\partial \pi_2}{\partial \theta_2} = \frac{\theta_1^2\{(\theta_1-\theta_2)-c[(1-2\phi)\theta_1+\phi\theta_2]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2}+$$

$$+\frac{\{(\theta_1-\theta_2)(4\theta_1-7\theta_2)+c[4\theta_1^2+\theta_1\theta_2-2\theta_2^2+\phi(5\theta_2(2\theta_1-\theta_2)-8\theta_1^2)]\}}{(4\theta_1-\theta_2)^3(\theta_1-\theta_2)^2} = \theta_2$$

action profile at $t=1$	θ_1^*	θ_2^*
$\{PC, PC\}$	$0.2533(1-c\phi)^2$	$0.0482(1-c\phi)^2$
$\{PD, PD\}$	$0.2533\frac{(1-c)^2}{\psi}$	$0.0482\frac{(1-c)^2}{\psi}$

Table 7: Quality outcomes - uncovered market, θ as a fixed and variable costs

					Nash Equilibrium, ψ–interval		
(c, ϕ)	ψ_1	ψ_2	ψ_3	ψ_4	PC, PC	PD, PC	PD, PD
(0.008,0.9)	0.9962	0.9962	0.99171	0.99173	0.9962–1	0.99173–0.9962	0–0.99173
(0.008,0.5)	0.9814	0.9814	0.9591	0.9598	0.9814–1	0.9598–0.9814	0–0.9598
(0.008,0.1)	0.9669	0.9669	0.9273	0.9289	0.9669–1	0.9289–0.9669	0–0.9289
(0.002,0.9)	0.9991	0.9991	0.9979	0.9979	0.9991–1	0.9979–0.9991	0–0.9979
(0.002,0.5)	0.9953	0.9953	0.9897	0.9897	0.9953–1	0.9897–0.9953	0–0.9897
(0.002,0.1)	0.9916	0.9916	0.9815	0.9816	0.9916–1	0.9816–0.9916	0–0.9816

Table 8: Nash equilibria in innovation adoption when quality is a fixed and variable cost

Claim 2 *When quality is a fixed and variable cost and* $\phi = 0.9$ *the innovation game has three types of equilibria:*
i. $\{PC, PC\}$ *if* $\frac{\bar{\phi}}{\psi} \in \quad [0.9, 0.9034]$*;*
ii. $\{PD, PC\}$ *if* $\frac{\bar{\phi}}{\psi} \in \quad]0.9034, 0.9075[$*;*
iii. $\{PD, PD\}$ *if* $\frac{\bar{\phi}}{\psi} \in \quad [0.9075, \infty]$*;*

In all $\{c, \phi\}$ configurations the same types of equilibria arise. In general, to identify the possible Nash equilibria in the innovation game, only parameters ψ_1, ψ_2, ψ_3 and ψ_4, as defined in (7)–(10), are relevant. Moreover, it is crucial to underscore that both ψ_3 and ψ_4 are lower than ψ_1 and ψ_2. This means that the ψ interval where the adoption of a product innovation is a dominant strategy for the high quality firm is larger than the ψ range which makes the same strategy dominant for the low quality firm.

The simulation analysis shows that the same types of equilibria always arise in every configuration; hence it is possible to generalize the results. The following Proposition states that only three types of equilibria can arise in the innovation game: two symmetric equilibria (where both firms adopt either a process innovation or a product innovation) and only one asymmetric equilibrium (where the high quality firm adopts a product innovation and the low quality firm a process innovation). Moreover it highlights that the determinant of these equilibria is index of the impact of these innovation on firm's costs function, i.e. the ratio between the costs saving effect and the quality effect.

Proposition 1 *The innovation game has the following Nash equilibria:*
i. (PC, PC) *if the index* $\frac{\overline{\phi}}{\psi}$ *belongs to the interval* $[\overline{\phi}, \frac{\overline{\phi}}{\psi_1}]$*;*
ii. (PD, PC) *if* $\frac{\overline{\phi}}{\psi} \in]\frac{\overline{\phi}}{\psi_1}, \frac{\overline{\phi}}{\psi_4}[$*;*
iii. (PD, PD) *if* $\frac{\overline{\phi}}{\psi} \in [\frac{\overline{\phi}}{\psi_1}, \infty]$.

Proof: See Appendix.

Proposition 1 highlights the interesting result that firms might choose asymmetrically between product and process innovation. Indeed firms have different incentives in adopting a product innovation, since introducing the latter becomes a dominant strategy for the high quality firm for a smaller level of the impact index than for the low quality firm. To obtain some intuition as to why a firm selling an high quality good has a propensity to favor product over process innovation, it is important to look at the so–called *direct* effect and *strategic* effect on firm i's profits of the introduction of a certain type of innovation.[17] Clearly the introduction of any type of innovation has a *positive* direct effect, since costs are lower, while the strategic effect is *negative* if one firm introduces a *process innovation* (the competitor will respond to a unit costs reduction by reducing its own price, thereby lowering the high quality firm's profits) and is *positive* if it adopts a *product innovation* (the competitor will react to an higher quality due to product innovation by increasing its price). Furthermore, since the high quality firm has higher fixed costs than the low quality firm, the direct effect of introducing a product innovation on its profits is greater than the direct effect of the same type of innovation on the low quality firm's profits. Hence the high

[17]The strategic effect is defined as the change on the profits of firm i through the change that the introduction of an innovation induces in the choices variables of the competitor, i.e. p_j and θ_j.

quality firm has a greater incentive to choose a product innovation than the low quality firm, so that as soon as the quality effect is sufficiently large it is the first to adopt a product innovation. A further increase in the quality effect leads also the low quality firm to adopt it, since the reduction in the fixed costs of quality becomes so large to overcome the benefits of a reduction in the unit costs of production. These different incentives provide an explanation of some real world stylized facts: high quality firms are usually the first to adopt a new product since they sell to consumers which receive high utility from quality, while low quality firms always try to reduce their prices since they sell to customers that mainly care about price levels, and a reduction in the unit costs of production is the way to achieve this goal.

Remark

As shown by Choi and Shin [1992], a problem arising in duopolistic models of vertical product differentiation and identical firms is that there exist two symmetric market equilibria in pure strategy (and one in mixed strategy). In one equilibrium firm i is the quality leader while in the other equilibrium is the quality follower.[18] The literature as labeled this change in quality leadership as *leapfrogging.* In this model leapfrogging does not occur since we assign labels to each firms: firm 1 is the *high* quality firm, while firm 2 is the *low* quality firm. In fact we do not face the problem of which is the firm selling the high quality good, but we investigate which type of innovation adopts the high quality firm and, simultaneously, the low quality firm.[19] In our analysis it does not matter *who* is the quality leader, but what type of innovation the leader (and the follower) adopts.[20]

[18] In order to ensure a unique equilibrium the investigation is either restricted to marginal analysis in the vicinity of one of these equilibria via technological constraints or quality' leadership is assigned at the beginning of the game and taken as given.

[19] For example, in the symmetric equilibrium (PC, PC) the quality leader chooses a process innovation, while the low quality firm has adopted the same type of innovation.

[20] The literature has also pointed out that leapfrogging might arise in case of a change of some parameters (costs, demand, ...) (see Motta, Cabrales and Thisse [1997]). Hence in this model we should check whether a change in the cost function due to innovation adoption should produce a change in the quality's leadership. Once again, assigning labels to each firm solve

$\{c, \phi\}$	pre–innovation	(PC, PC)	(PD, PC)	(PD, PD)
(0.008,0.9)	5.449	5.351 $\Downarrow$	5.764 $\Uparrow$ $(\psi = 0.9)$	5.180 $\Downarrow$ $(\psi = 0.5)$
(0.008,0.5)	5.449	5.180 $\Downarrow$	5.986 $\Uparrow$ $(\psi = 0.8)$	5.180 $\Downarrow$ $(\psi = 0.5)$
(0.008,0.1)	5.449	5.216 $\Downarrow$	5.868 $\Uparrow$ $(\psi = 0.8)$	5.173 $\Downarrow$ $(\psi = 0.4)$
(0.002,0.9)	5.183	5.187 $\Uparrow$	5.297 $\Uparrow$ $(\psi = 0.97)$	5.209 $\Uparrow$ $(\psi = 0.5)$
(0.002,0.5)	5.183	5.209 $\Uparrow$	5.843 $\Uparrow$ $(\psi = 0.85)$	5.205 $\Uparrow$ $(\psi = 0.5)$
(0.002,0.1)	5.183	5.241 $\Uparrow$	6.122 $\Uparrow$ $(\psi = 0.8)$	5.216 $\Uparrow$ $(\psi = 0.4)$

Table 9: Effects on the degree of vertical differentiation - quality as fixed costs

4 The effect on the degree of vertical differentiation

In this Section we show the impact of the equilibria in the innovation game presented before upon the degree of vertical differentiation, defined by $\frac{\theta_1^*}{\theta_2^*}$. The latter is a measure of the intensity of competition between the duopolists: an increase in the degree yields a reduction in the intensity of competition and vice versa. If quality is a fixed costs the changes in degree of vertical differentiation from the pre–innovation equilibrium to each possible post–innovation equilibrium are shown in Table 9, for each costs configurations considered in the numerical solutions.

These changes are mixed: if unit costs are high (i.e. $c = 0.008$) the degree decreases if firms take symmetric adoptions. For instance, if the costs saving effect is $\phi = 0.9$ the degree shrinks from 5.449 (the pre–innovation level) to 5.351 if firms introduce a process innovation or to 5.180 (when the quality effect is $\psi = 0.5$) if the two competitors adopt a product innovation.[21] However the degree always increases in case of different innovations. When instead the unit costs are low

this problem. Last, leapfrogging also means that the two firms exchange their market segments in quality terms, i.e. the low (high) quality firm becomes the high (low) quality one, given the quality done by the other firm. To be sure that our candidate equilibria are indeed robust to this type of leapfrogging we have then to check that, given θ_i^*, firm j has no incentives to leapfrog the rival firm and produces itself the highest quality. When we apply this procedure to our simulation described above, we get that no leapfrogging occurs.

[21]The value of the quality effect must lie in the interval where equilibrium (PD, PC) or equilibrium (PD, PD) arise.

($c = 0.002$) the degree of vertical differentiation always increases. The reason of these different changes in the degree is the U–shaped relationship between the unit costs of production and the degree: if the unit costs are very low (high), a further decrease leads to an increase (decrease) in the degree.

If quality is a fixed and also a variable cost the degree of vertical differentiation before the innovation adoption is

$$\frac{0.2533(1-c)^2}{0.0482(1-c)^2} = 5.2512 \tag{11}$$

If the equilibrium in the innovation game is (PC, PC) the new degree is

$$\frac{0.2533(1-\phi c)^2}{0.0482(1-\phi c)^2} = 5.2512 \tag{12}$$

i.e. the adoption of a process innovation by both firms does not change the intensity of competition. The same result holds if the equilibrium in the innovation game is (PD, PD), since the new degree is

$$\frac{0.2533\frac{(1-c)^2}{\psi}}{0.0482\frac{(1-c)^2}{\psi}} = 5.2512 \tag{13}$$

If instead the impact index is such that the two firms select (PD, PC) the changes in the degree, for the different configurations analyzed in the numerical solution, are shown in Table 10. Again if firms choose asymmetrically the intensity of competition is relaxed. Hence, since an increase in the degree of vertical differentiation is always achieved if firms introduce different types of innovation, thereby making the result of a strategic choice costs heterogeneity between competitors, it is possible to state the following.

Proposition 2 *Costs heterogeneity induced by strategic innovation adoption relaxes intensity of competition, while symmetric adoption of each type of innovation increases the intensity of competition.*

Proposition 2 points out the following interesting result: firms have an incentive to create costs heterogeneity since it relaxes the intensity of competition. Hence the existence of an efficiency gap may be the result of an endogenous rational choice rather than an exogenous assumption.

$\{c, \phi\}$	pre–innovation	(PD, PC)
(0.008,0.9)	5.2512	5.2546 $\Uparrow$ ($\psi = 0.995$)
(0.008,0.5)	5.2512	5.3297 $\Uparrow$ ($\psi = 0.96$)
(0.008,0.1)	5.2512	5.3913 $\Uparrow$ ($\psi = 0.93$)
(0.002,0.9)	5.2512	5.2549 $\Uparrow$ ($\psi = 0.998$)
(0.002,0.5)	5.2512	5.2701 $\Uparrow$ ($\psi = 0.99$)
(0.002,0.1)	5.2512	5.2738 $\Uparrow$ ($\psi = 0.985$)

Table 10: Effects on the degree of vertical differentiation - quality as fixed and variable costs

5 Conclusions

This paper investigates a duopoly model of vertical differentiation where firms simultaneously select whether to adopt a process innovation or a product innovation and compete in prices. The two innovations have different impacts on firm's efficiency, identified by a costs saving effect (process innovation) and by a quality effect (product innovation): since firms might adopt either one type of innovation or the other, the ratio between the costs saving effect and the quality effect can be defined as the impact index of the two innovations on firm's efficiency. The analysis has produced the following results: First, three equilibria in the innovation game may arise: two symmetric (where both firms choose either a process or a product innovation) and one asymmetric (where the high (low) quality firm selects a product (process) innovation. Second, the determinant of these equilibria is the size of the impact index: the greater the impact index (i.e. the greater is the quality effect) the more likely is that both firms adopt a product innovation. However, since the high quality firm has more incentives to sell goods with higher quality, it is the first to adopt a product innovation, so that there exists an interval of the impact index where firms choose asymmetrically. Third, the above equilibria have different effects on the degree of vertical differentiation: specifically, the latter increases only if firms adopt different types of innovation, i.e. if they induce costs heterogeneity. Hence the intensity of competition is not relaxed by the symmetric adoption of each type of innovation but through the creation of an efficiency gap.

ψ range	Firm 1	Firm 2	Nash equil.
$\psi_1 \leq \psi \leq 1$	$\{(PC, PC); (PC, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PC, PC)
$\psi_2 < \psi < \psi_1$	$\{(PD, PC); (PC, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PD, PC)
$\psi_3 \leq \psi \leq \psi_2$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PD, PC)
$\psi_4 < \psi < \psi_3$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PD); (PD, PC)\}$	(PD, PC)
$0 \leq \psi \leq \psi_4$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PD); (PD, PD)\}$	(PD, PD)

Table 11: Best replies and Nash equilibria if $\psi_1 > \psi_2$ and $\psi_3 > \psi_4$

6 Appendix

Proof of Proposition 1.

Since ψ_1 and ψ_2 are higher than ψ_3 and ψ_4 we have to consider only four possible cases: (1) $\psi_1 > \psi_2$ and $\psi_3 > \psi_4$, (2) $\psi_2 > \psi_1$ and $\psi_3 > \psi_4$, (3) $\psi_1 > \psi_2$ and $\psi_4 > \psi_3$, (4) $\psi_2 > \psi_1$ and $\psi_4 > \psi_3$.

Case (1). The best replies, and the related Nash equilibria, according to the different ψ levels are shown in Table 11. The symmetric equilibria where both firms adopt a process innovation arises when $\frac{\overline{\phi}}{\psi} \in [\overline{\phi}, \frac{\overline{\phi}}{\psi_1}]$. The asymmetric equilibrium (PD, PC) for $\frac{\overline{\phi}}{\psi} \in]\frac{\overline{\phi}}{\psi_1}, \frac{\overline{\phi}}{\psi_4}[$. The symmetric equilibrium where both firms adopt a product innovation is the solution of the innovation game for $\frac{\overline{\phi}}{\psi} \in [\frac{\overline{\phi}}{\psi_4}, \infty]$.

Case (2). Table 12 displays the Nash equilibria. If we compare this case with case (1) the asymmetric equilibrium in the innovation game arises in a smaller $\frac{\overline{\phi}}{\psi}$ range than in the former case. The different Nash equilibria are the solution of the innovation game for the same range of the impact index shown in case (1).

Case (3). The ψ ranges which lead to the different Nash equilibria are shown in Table 13. As before, we have different ranges where the three possible types of equilibria arise. Moreover, the ranges of $\frac{\overline{\phi}}{\psi}$ which sustain the different equilibria are the same as those shown in case (1).

Case (4). Table 14 reports the Nash equilibria for the last case. The same $\frac{\overline{\phi}}{\psi}$ ranges as case (1) apply also to this case.

□

ψ range	Firm 1	Firm 2	Nash equil.
$\psi_2 < \psi \leq 1$	$\{(PC, PC); (PC, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PC, PC)
$\psi_1 \leq \psi \leq \psi_2$	$\{(PC, PC); (PD, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PC, PC)
$\psi_3 \leq \psi < \psi_1$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PD, PC)
$\psi_4 < \psi < \psi_3$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PD); (PD, PC)\}$	(PD, PC)
$0 \leq \psi \leq \psi_4$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PD); (PD, PD)\}$	(PD, PD)

Table 12: Best replies and Nash equilibria if $\psi_2 > \psi_1$ and $\psi_3 > \psi_4$

ψ range	Firm 1	Firm 2	Nash equil.
$\psi_1 \leq \psi \leq 1$	$\{(PC, PC); (PC, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PC, PC)
$\psi_2 < \psi < \psi_1$	$\{(PD, PC); (PC, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PD, PC)
$\psi_4 < \psi \leq \psi_2$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PD, PC)
$\psi_3 \leq \psi \leq \psi_4$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PC); (PD, PD)\}$	(PD, PD)
$0 \leq \psi < \psi_3$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PD); (PD, PD)\}$	(PD, PD)

Table 13: Best replies and Nash equilibria if $\psi_1 > \psi_2$ and $\psi_4 > \psi_3$

ψ range	Firm 1	Firm 2	Nash equil.
$\psi_2 < \psi \leq 1$	$\{(PC, PC); (PC, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PC, PC)
$\psi_1 \leq \psi \leq \psi_2$	$\{(PC, PC); (PD, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PC, PC)
$\psi_4 < \psi < \psi_1$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PC); (PD, PC)\}$	(PD, PC)
$\psi_3 \leq \psi \leq \psi_4$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PC); (PD, PD)\}$	(PD, PD)
$0 \leq \psi < \psi_3$	$\{(PD, PC); (PD, PD)\}$	$\{(PC, PD); (PD, PD)\}$	(PD, PD)

Table 14: Best replies and Nash equilibria if $\psi_2 > \psi_1$ and $\psi_4 > \psi_3$

ψ	θ_1	θ_2	π_1	π_2
1	0.2523	0.0476	0.0225	0.00017
0.9	0.2799	0.0486	0.0259	0.00019
0.8	0.3145	0.0496	0.0302	0.0002
0.7	0.359	0.0506	0.0357	0.00022
0.6	0.4183	0.0517	0.043	0.00024
0.5	0.5014	0.0528	0.0533	0.00026
0.4	0.6262	0.054	0.0689	0.00028
0.3	0.8342	0.055	0.0948	0.0003
0.2	1.2506	0.056	0.1468	0.00032
0.1	2.500	0.057	0.303	0.00034

Table 15: Firms' profits when (PD, PC), $c = 0.008$, $\phi = 0.9$

	Firm 1 innovation adoption							
	PC				PD			
ψ	θ_1	θ_2	π_1	π_2	θ_1	θ_2	π_1	π_2
1	0.25207	0.0458	0.0234	0.000026	0.25209	0.0462	0.023	0.00004
0.9	0.2527	0.0507	0.0227	0.00014	0.2803	0.0524	0.0256	0.00018
0.8	0.2534	0.0562	0.0218	0.00036	0.3156	0.0598	0.0289	0.00036
0.7	0.2544	0.0699	0.0208	0.00046	0.3609	0.069	0.0331	0.0006
0.6	0.2558	0.0699	0.0195	0.00068	0.4213	0.081	0.0388	0.00094
0.5	0.2577	0.0788	0.018	0.00096	0.5058	0.098	0.0469	0.0014
0.4	0.2603	0.0896	0.016	0.0013	0.6326	0.1223	0.059	0.0021
0.3	0.2641	0.1031	0.0134	0.0018	0.8437	0.163	0.079	0.0034
0.2	0.2699	0.1266	0.0099	0.0025	1.266	0.2438	0.12	0.0059
0.1	0.279	0.144	0.0046	0.0034	2.532	0.4856	0.242	0.0135

Table 16: Firms' profits when firm 2 chooses PD, $c = 0.008$, $\phi = 0.9$

ψ	θ_1	θ_2	π_1	π_2
1	0.2521	0.0458	0.0234	0.000026
0.9	0.2527	0.0507	0.0227	0.00014
0.8	0.2534	0.0562	0.0218	0.00029
0.7	0.2544	0.0625	0.0208	0.00046
0.6	0.2558	0.0699	0.0195	0.00068
0.5	0.2577	0.0788	0.018	0.00096
0.4	0.2603	0.0896	0.016	0.0013
0.3	0.2642	0.1031	0.0134	0.0018
0.2	0.2699	0.1206	0.0099	0.0025
0.1	0.279	0.144	0.0046	0.0034

Table 17: Firms' profits when (PC, PD), $c = 0.008$, $\phi = 0.9$

	Firm 2 innovation adoption							
	PC				PD			
ψ	θ_1	θ_2	π_1	π_2	θ_1	θ_2	π_1	π_2
1	0.2523	0.0476	0.0225	0.00017	0.2521	0.0463	0.023	0.00004
0.9	0.2799	0.0486	0.0259	0.00019	0.2803	0.0524	0.0256	0.00018
0.8	0.3145	0.0496	0.0302	0.00021	0.3156	0.0598	0.0289	0.00036
0.7	0.359	0.0506	0.0357	0.00022	0.3609	0.069	0.0331	0.0006
0.6	0.4183	0.0517	0.0430	0.00024	0.4213	0.081	0.0389	0.00094
0.5	0.5014	0.0528	0.0533	0.00026	0.5058	0.0976	0.0469	0.0014
0.4	0.6262	0.054	0.0689	0.00028	0.6326	0.1223	0.0590	0.0021
0.3	0.8342	0.0551	0.0948	0.0003	0.8437	0.163	0.0793	0.0034
0.2	1.251	0.0563	0.1468	0.00032	1.266	0.2439	0.12	0.0059
0.1	2.5	0.0575	0.303	0.00034	2.533	0.4856	0.242	0.0135

Table 18: Firms' profits when firm 1 chooses PD, $c = 0.008$, $\phi = 0.9$

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